Summer 1976

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Plea Bargaining, Decision Theory, and Equilibrium Models: Part I

STUART S. NAGEL* & MARIAN NEEF**

I. BASIC MATTERS

Plea bargaining refers to the negotiations between a prosecutor and a defense lawyer or defendant over the prosecutor's charge or over the sentence which the prosecutor will recommend to the judge in return for a plea of guilty. A high percentage of the disputes that enter into the judicial process at either the criminal or civil complaint stage are resolved by bargaining among the lawyers or litigants rather than by a trial.1

Most of the literature on plea bargaining has consisted of descriptions or evaluations of the legal rules governing the relation between prosecutors and defendants2 or anecdotal descriptions of plea bargaining incidents.3 The legal-evaluative literature emphasizes that if a prose-

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1 In the Wisconsin county studied by Newman, he found: only 6 percent of all felonies went to trial; 40 percent involved cases where the defendant originally entered a not guilty plea which he subsequently changed to a guilty plea largely because of plea bargaining; and 54 percent involved cases where the defendant originally entered a guilty plea which may or may not have been a negotiated guilty plea. Of the 94 percent who pleaded guilty, 60 percent did so with bargaining, and 40 percent without bargaining. Newman, Pleading Guilty for Considerations: A Study of Bargain Justice, 46 J. Crim. L.C. & P.S. 780 (1956). In New York City, only 2 percent of all negligence claims are terminated by trial, with 98 percent terminated by out-of-court settlements or withdrawals. Franklin, Chanin, & Mark, Accidents, Money and the Law: A Study of the Economics of Personal Injury Litigation, in DOLLARS, DELAY AND THE AUTOMOBILE VICTIM: STUDIES IN REPARATION FOR HIGHWAY INJURIES AND RELATED COURT PROBLEMS 25, 39-40 (Walter E. Meyer Research Institute of Law ed. 1968).

2 See, e.g., ABA Project on MINIMUM STANDARDS FOR CRIMINAL JUSTICE, STANDARDS RELATING TO PLEAS OF GUILTY (1967) and the references cited therein.

3 See, e.g., J. BOND, PLEA BARGAINING AND GUILTY PLEAS (1975); M. MARCUS & R. WHEATON, PLEA BARGAINING: A SELECTED BIBLIOGRAPHY (1976); D. NEWMAN, CONVICTION:
cutor promises a defendant a lighter sentence for pleading guilty, then
a guilty plea under those circumstances does not constitute an involun-
tary confession, but rather a generally useful method for reducing court
congestion. The anecdotal literature often mentions instances where the
system is used to take advantage of hard-pressed defendants or prose-
cutors, or where other socially undesirable results occur.

It is the purpose of this article to discuss plea bargaining mainly
from the perspective of what actually tends to occur rather than from
the perspective of what the law or other evaluators say should occur.
This empirical perspective avoids human interest anecdotes in order to
concentrate on the general essence of the plea bargaining transaction.
By doing so, a descriptive model of plea bargaining will be developed
that has enough empirical validity and generality to be able to answer
questions about the effects of judicial system changes on the likelihood
and the level of plea bargaining settlements.

System changes, in this context, refer to the effects of changes in
the degree of pretrial release, provision of counsel to the indigent, judi-
cial delay, mutual discovery, exclusion of illegally seized evidence, aboli-
tion of capital punishment, or other changes that affect the litigation or
settlement costs of the parties, the probability of conviction, or affect
the severity of sentences. A model that is capable of predicting the effects
of changes like those in the criminal justice system seems worth pur-
suing in order to better plan and understand the operations of the
criminal justice process.

For example, in the early 1960's partly as a result of studies by
the Vera Institute, the New York criminal courts increased the extent
to which defendants were released prior to trial. This policy, admin-
istered under a system of quantitative screening and pretrial notifica-
tion, simultaneously kept the costs of re-arresting non-appearing de-
fendants at the former cost levels in spite of the higher release rate,
while reducing the costs of imprisonment to the defendant and society. The
costs that were reduced included jail maintenance, lost gross national product,
and bitterness generated by imprisonment for a complaint that is eventually dismissed or
withdrawn, at least for those released who would have formerly been held.
PLEA BARGAINING

was reduced since many defendants were no longer suffering the high litigation cost of being held in jail prior to trial if they refused to settle. If the new congestion causes those who are in jail to be subjected to longer delay to have their cases tried, then the number of pretrial jail inmates may actually increase even though the percent being sent to jail to await trial has decreased. If the system had a better model of its operations, that effect could have been anticipated and prepared for mainly by having the prosecutors offer a greater discount on their settlement offers.

A model designed to describe a social process like plea bargaining consists of a set of statements about the relations between certain inputs, causes, or predictor variables on the one hand, and certain outputs, effects, or predicted variables on the other hand. A good descriptive model is one in which (1) the conclusions logically follow from the premises, (2) the premises conform to empirical reality, (3) the conclusions have broadness in time, geography, and abstractness, (4) the conclusions help explain why things happen and how one might more effectively achieve given goals, (5) the relations between the variables are capable of being expressed with some measureability so that the model can be objectively applied, and (6) the total structure is simple and understandable but captures the essence of a complex phenomenon.

The plea bargaining model presented in this article tries to achieve those goals. The only assumptions the model makes are that defendants normally want to minimize their sentences and their likelihood of being convicted, and prosecutors normally want to maximize the sentences, within legal and ethical constraints, and the likelihood of obtaining a conviction. The model is largely based on the concepts and methods of decision theory and equilibrium modeling which have been used by economists and operations researchers to study bargaining and exchange relations among buyers and sellers, unions and management, and among competing business firms.

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6 On social science models in general, especially those that can be mathematically formulated, see H. Blalock, THEORY CONSTRUCTION: FROM VERBAL TO MATHEMATICAL FORMULATIONS (1969); M. Greenberger, MODELS IN THE POLICY PROCESS (1976); A GUIDE TO MODELS IN GOVERNMENTAL PLANNING AND OPERATIONS (S. Gass & R. Sisson eds. 1973); C. Lave & J. March, AN INTRODUCTION TO MODELS IN THE SOCIAL SCIENCES (1975); R. Singleton & W. Tyndall, GAMES AND PROGRAMS: MATHEMATICS FOR MODELING (1974).

7 For further detail on the assumptions of this model, see Section IV-C1 in Nagel & Neef, Plea Bargaining, Decision Theory, and Equilibrium Models, Part II, 52 IND. L.J. — (Fall 1976) [forthcoming] [hereinafter cited as Part II].
Decision theory is the mathematical tool that determines which combination of various available decisions should be reached in order to maximize given goals in light of probabilistic or uncertain events. If those uncertain events are the actions of decision makers with conflicting interests, a gaming-type of decision theory may be involved. If those uncertain events are the actions of nature or a non-person, e.g., the weather, or the actions of some person or group who is not in conflict with the decisionmaker, e.g., the jury or bench trial in plea bargaining, then the kind of decision theory involved is often referred to as a game against nature or simply as decision theory.\(^8\)

An equilibrium model can be defined as a set of quantitative statements that describe the behavior of objects such that if the statements are true, the objects will tend to move toward a given equilibrium whenever they are displaced from it.\(^9\) Equilibrium models can be either static or dynamic. With a static model, one can say, given certain assumptions and starting data, where the objects will end. With a dynamic model, one can also say what steps will be involved in moving over time from the starting point to the equilibrium point.\(^10\)

In addition to the main purpose of this article of presenting a mathematical model that captures the essence of the plea bargaining process for testing hypotheses and integrating findings, the article will also develop a model that can provide insights to legal practitioners for improving their bargaining techniques and, especially that can provide insights to legal policymakers for improving the operations of the criminal justice system.\(^11\) In addition, it is hoped that the article will clarify the process

\(^8\) For further literature on decision theory and related game theory, see R. MACK, PLANNING ON UNCERTAINTY: DECISION MAKING IN BUSINESS AND GOVERNMENT ADMINISTRATION (1971); H. RAFFA, DECISION ANALYSIS: INTRODUCTORY LECTURES ON CHOICES UNDER UNCERTAINTY (1968); A. RAPPAPORT, TWO-PERSON GAME THEORY: THE ESSENTIAL IDEAS (1966) [hereinafter cited as RAPPAPORT]; S. RICHMOND, OPERATIONS RESEARCH FOR MANAGEMENT DECISIONS 501-560 (1968) [hereinafter cited as RICHMOND].

\(^9\) For further literature on equilibrium models, see M. BRENNAN, PREFACE TO ECONOMETRICS 199-250 (1973) [hereinafter cited as BRENNAN]; J. CROSS, ECONOMICS OF BARGAINING (1969) [hereinafter cited as CROSS]; P. NEWMAN, THE THEORY OF EXCHANGE (1965).

\(^10\) For further detail on the usefulness of this model, see Part II, supra note 7, at Section IV-B.

\(^11\) Numerous concrete examples could be given of other applications of mathematical models to the legal process. See Part II, supra note 7, at Sections IV-C2 and IV-C3. Those applications include probabilistic decision theory applications (e.g. where an arraignment judge is trying to decide whether to release a defendant prior to trial contingent on the probability of the defendant appearing in court) and bargaining applications (e.g. where two contract negotiators are trying to decide on contract terms). Those applications also include equilibrium models with an optimum mix orientation (e.g. finding an optimum mix between law reform and case handling in the OEO Legal Services Program) and equilibrium models with an optimum level orientation (e.g. the optimum jury size problem). Before one can understand or appreciate the variety of applications, it is probably best
of applying decision theory and equilibrium models to certain aspects of law and the legal process so that others will be better able to use those concepts and methods in studying other aspects of law, policy, politics, and the legal process.

II. Decision Theory Applied to the Defendant and the Prosecutor

The quantitative statements that serve as the basis for the static and dynamic equilibrium models of the plea bargaining process utilized in this article are derived from decision theory. In presenting the defendant's and the prosecutor's perspective of the decisionmaking process, hypothetical but realistic cases will be used. Throughout the article, methods of obtaining survey and other data will be mentioned, as well as methods of applying such data to test the theories presented.

A. The Payoff Matrices as Perceived by the Bargainers

1. Interpreting the Matrices

TABLE I shows how a defendant and a prosecutor in a hypothetical case each views the most likely sentence if (a) the defendant pleads guilty before a judge in a non-negotiated plea when his probability of being convicted (or PC) is relatively near zero, (b) the defendant pleads guilty before a judge when in a non-negotiated plea the probability of his being convicted is relatively near 1.0, (c) the defendant goes to trial when his probability of conviction is zero, and (d) the defendant goes to trial when his probability of conviction is 1.0. The cell entries show likely sentences in terms of years for a crime that allows for at least as much as 10 years in prison, which would mean a major felony case.\textsuperscript{12} The prosecutor and the defense more often bargain over the

\textsuperscript{12} One advantage of working with a major felony case is that all the payoff cells can be measured in years in prison. With a lesser crime, cell a might yield probation for a long time, probation for a short time, a suspended sentence, or a fine. If all four cells cannot be measured in the same units like years or portions of years, then one may have to express the cell payoffs in relative units or rank orders rather than absolute units. It would, however, be arbitrary to equate guilty plea probation with the zero years from acquittal associated with cell c. See note 27 infra; Part II, supra note 7, at Section IV-C1, on handling a relative units payoff matrix.

In order to measure all four cells in the same units where one is dealing with payoffs that relate to probation, fines, and jail sentences, one can resort to asking a defendant, defense counsel, or prosecutor the following type of questions: How much would you have to be offered to make it worth your while to plead guilty and receive a one year sentence for a certain crime that you did not commit? How much would you have to be offered to make it worth your while to plead guilty and receive two years probation for a certain crime that you did not commit? The questions could be extended to any crime and any type sentence. The answers would indicate in dollars rather than index numbers how the
TABLE 1. THE PAYOFF MATRICES AS PERCEIVED BY A DEFENDANT AND A PROSECUTOR

1A. A DEFENDANT'S PAYOFF MATRIX

<table>
<thead>
<tr>
<th>Alternative Decisions of D</th>
<th>Probability of D Being Convicted (PC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>D Pleads Guilty before a Judge</td>
<td>a</td>
</tr>
<tr>
<td>without Bargain (Alt. #2)</td>
<td>4</td>
</tr>
<tr>
<td>D Goes to Trial (Alt. #1)</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Cells indicate likely sentences (LS) in years as perceived by a hypothetical defendant (D).

1B. A PROSECUTOR'S PAYOFF MATRIX

<table>
<thead>
<tr>
<th>Alternative Decisions of D</th>
<th>Probability of D Being Convicted (PC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>D Pleads Guilty before a Judge</td>
<td>a</td>
</tr>
<tr>
<td>without Bargain (Alt. #2)</td>
<td>3</td>
</tr>
<tr>
<td>D Goes to Trial (Alt. #1)</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Cells indicate likely sentences (LS) in years as perceived by a hypothetical prosecutor (P).

crime with which the defendant should be charged rather than what sentence the prosecutor should recommend since the prosecutor has more control over the charge than the sentence. Different charges, however, can be translated into sentences by thinking in terms of the average, maximum, or minimum sentence for the charge or a range of sentences on the charge.\(^\text{13}\)

respondent values the sentences relative to each other. The method, however, does not provide for an interpersonal unit or measure of satisfaction since the value of a dollar varies from individual to individual depending partly on how many dollars he already has.

\(^{13}\) An attempt has been made to use realistic numbers in the cell entries, but the relative size or even rank order of the cells has no essential bearing on the meaningfulness of the decision theory equilibrium model. See Section II-A2 infra for a discussion of various
Table 1 shows that both the defendant and the prosecutor perceive the most severe sentence of the four payoff categories as likely to be received if the defendant goes to trial when the probability of conviction is high, and the least severe sentence to be received when the defendant goes to trial with a low probability of conviction. Thus comparing row \( ab \) and row \( cd \), the defendant has more to lose and more to gain by going to trial than by a non-negotiated guilty plea before a judge. If the conviction probability is zero, the defendant receives a longer sentence by pleading guilty than by going to trial where he thinks he will be acquitted. The judge, in taking a guilty plea, will assume the defendant is guilty, even if the probability of conviction is low due to the weakness of the evidence. If the conviction probability is 1.0, the defendant receives a shorter sentence by pleading guilty than by going to trial where he thinks he will be convicted. The judge in effect rewards the defendant (1) for saving the court time and money by not going to trial, (2) for showing contriteness through a guilty plea confession, and (3) for saving the victim and the witnesses from having to testify. It is an exceptional situation for a jury on conviction to give a lower sentence than the judge taking a plea gives since few states allow juries to participate in sentencing.

Table 1 also shows that both the defendant and the prosecutor perceive that when the conviction probability is low, the defendant will receive a lower sentence on a guilty plea before a judge than when the conviction probability is high. This is so because the stronger the evidence is against a defendant, the more confident the judge will be about methods for predicting likely sentences or cell payoffs in given cases. These methods can also be used to translate charges into likely sentences while considering other circumstances in the case.

14 For the sake of simplicity, the table does not make any distinction between going to a bench trial or going to a jury trial. Likewise, it does not distinguish between pleading guilty before one judge or another judge. The trial row of the payoff matrix should show the payoff of whichever is lower between a bench trial and a jury trial. Likewise, the pleading row should show the payoff for whichever judge is likely to give the lowest sentence where "judge shopping" is possible or the mean sentence where random assignment is used. Otherwise, it should show the likely payoffs for the judge who has been assigned to the case. Additional rows could be added to distinguish between bench trial and jury trial and between pleading before one judge or another judge, but doing so in this context would serve no useful purpose although a decision theory payoff matrix often has more than two rows or two alternative decisions.

The defendant should work with the lowest payoffs available to him in implicitly preparing his payoff matrix since the matrix is designed to determine the defendant's fallback position or best alternative position if plea bargaining breaks down. To be more exact, the defendant should work with the combination of conviction probabilities and payoffs that are the lowest. Thus, a bench trial row should be used rather than a jury trial row if the combination of bench trial conviction probabilities and sentencing payoffs are lower than the combination of jury trial conviction probabilities and sentencing payoffs. The prosecutor should do likewise since the prosecutor's payoff matrix is designed to determine the worst
the defendant's guilt and the more severe the judge is likely to be. Therefore, comparing column $ac$ and column $bd$, the defendant will receive a more severe sentence if his probability of conviction is high rather than low regardless whether he pleads guilty or goes to trial.

Comparing Table 1A and Table 1B, it is apparent that the hypothetical defendant perceives his likely payoff sentences as generally being more severe than the prosecutor perceives them. This may be because the defendant is more aware than the prosecutor of his own guilt or of aggravating circumstances, and defendants usually are guilty as indicated by high conviction rates. On the other hand, the defendant may perceive lower payoff sentences than the prosecutor because of wishful thinking or because the defendant is aware of his own innocence, or he is aware of mitigating circumstances. The decision theory equilib-
rium model, however, need not presuppose anything with regard to the relative perceptions of the defendant and the prosecutor\(^ {18} \) in order to be meaningfully applied.\(^ {19} \)

To clarify the terminology, four kinds of payoff cells must be distinguished. In this context, the reference is not to the fact that there are four payoff cells, namely \(a\), \(b\), \(c\), and \(d\) in \textit{Table 1}. Rather, the reference is to the fact that the payoff cells can be those perceived by the defendant, by the prosecutor, by an omniscient being, or by an omnibenevolent being. \textit{Table 1A} and \textit{Table 1B} show the payoff cells as perceived by the defendant and the prosecutor respectively. To the extent both sides are represented by experienced, knowledgeable attorneys, those payoff cells should be reasonably accurate. The true payoff cells, which are unknowable, indicate exactly what sentence the defendant will receive. Those true payoff values are known with certainty only by one with omniscient powers, although the next section will deal with how

\(^ {18} \text{In most of this article, the plea bargaining process is referred to as involving basically two bargainers, the prosecutor and the defendant. However, occasionally it will be recognized that the prosecutor may not be a single individual, but rather a number of assistant prosecuting attorneys who have differing perceptions of the probability of conviction and the sentencing payoffs. When the prosecutor's perception is discussed, it will be in reference to any collective perception of the prosecuting attorneys involved in the case. Likewise, the defense side does not always consist of one mind, but rather of a defendant and sometimes more than one defense attorney. Thus, the defense bargainer is also a collective entity where the perceptions of the defense attorney normally counts for substantially more than the perceptions of the defendant although how their input varies is an empirical question. Sometimes their goals may even conflict (or at least not be the same) with regard to the importance of minimizing the sentence and the litigation costs. There may thus be a separate bargaining game occurring between the defendant and his attorney, but modeling that bargaining process is a subject for a separate article.}

\(^ {19} \text{As another alternative to using the sentence years approach in \textit{Table 1}, one could determine for each cell how much it differs from the worst cell as a measure of benefits, and subtract from each cell the opposite cell in the same column as a measure of opportunity costs. One could likewise determine for each cell how much it differs from the best cell as a measure of costs, and subtract each cell from the opposite cell in the same column as a measure of opportunity benefits. Opportunity costs indicate what one is sacrificing by choosing an alternative decision at a given probability of conviction, whereas opportunity benefits indicate what one is gaining. These kinds of benefit minus cost calculations, however, result in units that have no meaning in the real world context of plea bargaining. In that real world, the defendant sees no benefits from being sentenced, only costs which he wants to minimize. Likewise, the prosecutor sees no costs from the \textit{issuance} of a sentence, only benefits which he wants to maximize subject to constraints. There may be costs to the prosecutor involved in obtaining or enforcing the sentence, but not in the sentence itself. The prosecutor is like a seller seeking as high a price as possible, and the defendant is like a buyer, seeking as low a price as possible.}
prosecutors, defendants, and social scientists can try to estimate the true payoff cells. The fourth set of payoff cells are those which indicate what sentences the defendant deserves to receive in situations \(a, b, c,\) and \(d.\) Those "just" payoff values are quite subjective and known, if at all, only by an omnibenevolent being who has a set of values that represent the right set of values, assuming of course that some right set of values exists. The mathematical model, however, does not require the user—whether a practicing lawyer, a legal scholar, a policymaker, or some other type of user—to know what the empirically true or the normatively just payoff cells actually are in order to apply the model.

2. Deriving the Matrices

A defendant, prosecutor, or social scientist must be able to determine what the sentences are likely to be for each of the four payoff cells. Cell \(c\) is logically always 0 if the cells are expressed exclusively in sentence time units. If stigma and money costs are also included, the defendant might suffer some adverse payoffs even if acquitted on trial.\(^{20}\)

Cell \(d\) can be figured by determining the average sentence received in all or a sample of the cases known to the defense lawyer, the prosecutor, or the docket files in which the defendant went to trial and was convicted of a charge similar to the one in our hypothetical case. If the defendant was convicted, then the evidence must have indicated close to a 1.0 probability of conviction, because defendants can theoretically only be convicted where there is no reasonable doubt which means roughly more than a .95 conviction probability. Cell \(d\) will be close to, but probably beneath the statutory maximum, since convicted felons seldom receive the maximum possible sentence.

The values for cells \(a\) and \(b\) are harder to determine. If the average sentence is used for all cases in which the defendant did plead guilty on a charge similar to one in the hypothetical case, then both cell \(a\) and cell \(b\) would have the same value, probably contrary to empirical reality. As a rule of thumb, one-third of the average trial conviction figure of cell \(d\) could be used in cell \(a,\) and two-thirds of cell \(d\) could be used in cell \(b.\)\(^{21}\) To avoid rules of thumb, knowledgeable prosecutors and defense

\(^{20}\) Likewise, the prosecutor might gain some positive payoffs from an acquittal if the trial serves to frighten other perceived social deviants. The prosecutor might also suffer adverse payoffs from a conviction via lost office resources which were devoted to the trial.

\(^{21}\) There is now available in the archives of the Inter-University Consortium for Political Research at Ann Arbor a sample of 11,256 criminal cases from across the country for the early 1960's from which one can derive average sentences to various crimes when the defendant (1) pleads guilty to a lesser charge, implying a negotiated plea; (2) pleads guilty to the original charge, implying a non-negotiated plea, or (3) has a trial and is found
attorneys could be asked, as part of a mailed questionnaire or selective interviewing, for how long would judges be likely to sentence the defendant when even though the probability of conviction is extremely low, the defendant pleads guilty and the judge believes him to be so. Likewise, they could also be asked for how long would the sentence run when the probability of conviction is extremely high and the defendant pleads guilty. Through such a questionnaire or interviewing approach, a rule of thumb might be refined by asking those knowledgeable persons what percent of cell \( d \) does the cell \( a \) sentence and the cell \( b \) sentence tend to represent and then averaging those responses to get mean responses for cell \( a \) and for cell \( b \). As a more statistically sophisticated approach to determining the likely sentences of cells \( a \), \( b \), and \( d \), a regression analysis could be used to predict sentences from the probability of conviction.

For example, in the subset of murder cases, when the defendants pleaded guilty to a lesser degree of murder, they averaged 7 years, figuring a life sentence at 20 years. When the murder defendants pleaded guilty to the original charge, their sentences averaged 17 years. When the murder defendants were found guilty in a trial, they averaged 14 years.

One obvious defect in this approach to obtaining real rather than hypothetical numbers for the payoff cells is the fact that the cases involving a guilty plea to the original charge may be more (or less) heinous than the cases which go to trial, and thus the unbargained guilty plea cases should involve a higher (or lower) average sentence than the guilty verdict cases because they are different cases. What is needed is to determine from knowledgeable experts how a case or case type is likely to be sentenced if it goes to trial, and how the same case is likely to be sentenced if the defendant pleads guilty without a bargain. If we apply the rule of thumb approach, we can say (1) the average murder case in the United States gets a sentence of 14 years if tried to conviction, (2) two-thirds of that or 9 years if pleaded guilty when a guilty verdict would have been received, and (3) 5 years or one-third of the 14 if pleaded guilty when an acquittal verdict would have been received.

A regression analysis might involve a linear prediction equation of the form \( L = A + B(PC) \) where \( A \) is zero since that is the likely sentence when \( PC \) is zero, and \( B \) is the number of units that \( LS \) changes when \( PC \) changes one unit. One such prediction equation would be needed for (1) all trial cases for each crime category and for (2) all guilty plea cases for each crime category. An alternative prediction equation might be \( LS = A + B_1(PC) + B_2(CS) + B_3(MR) \) subject to the condition that when \( PC \) equals 0, \( LS \) equals 0. In this equation, \( CS \) is a measure of crime severity based on the sentences provided in the statutes, \( MR \) refers to the method of resolution with a 1 for going to trial and a 2 for pleading guilty, and the \( B \)'s represent the weights by which the variables have to be
3. Translating the Matrices into Satisfaction Units

The likely sentences shown in Table 1 can be translated into dissatisfaction or satisfaction units to reflect the principles of diminishing disutility and utility. It is reasonable to assume that the defendant receives dissatisfaction from being sentenced although the model can be modified to include an unusual defendant who wants to be martyred, is masochistic, or has other reasons for wanting a longer sentence. If it is assumed that defendants receive dissatisfaction from being sentenced, it is also reasonable to assume that the incremental dissatisfaction increases at a decreasing rate with each incremental year he receives as part of his sentence. This is likely to be so by virtue of the general principle of diminishing marginal disutility which says that the more of a bad thing one has, the less dissatisfaction one gets out of each incremental unit.

Likewise, it is reasonable to assume that the prosecutor receives satisfaction from obtaining a sentence of a defendant that he believes is guilty, although the model can be modified to include the behavior of an unusual prosecutor who is seeking to enable a given defendant to receive as light a sentence as possible. Presumably, no ethical prosecutor seeks to obtain a sentence or to plea bargain with a defendant that he believes is innocent. It is also reasonable to assume that the incremental satisfaction to the prosecutor increases at a decreasing rate with each incremental year he obtains as part of the sentence to the point where diminishing absolute, not just marginal, utility sets in when the prose-

The symbols A and B (rather than a and b) are used throughout this article to represent regression coefficients so as to avoid confusion with the cell payoffs. Perhaps the cell payoffs should have been designated with capital letters since it is customary to designate unstandardized regression coefficients with lower case letters. For further detail on using linear regression analysis to predict sentences or other case outcomes, see Nagel, *Predicting Court Cases Quantitatively*, 63 MICH. L. REV. 1411 (1965); Tanenhaus et al., *The Supreme Court's Certiorari Jurisdiction: Cue Theory*, in *JUDICIAL DECISION MAKING* (G. Schubert ed. 1963). As the probability of conviction goes up, the likely sentence also goes up at what is probably a fairly constant rate although empirical data would be useful to confirm that. As crime severity goes up, however, the likely sentence may go up but at a decreasing rate. The best fitting curve might thus involve using the logarithm of CS instead of CS in the above equation. On nonlinear regression analysis, see J. Guilford, *PSYCHOMETRIC METHODS* 43-78 (1954). The nonlinear diminishing-returns relation between likely sentence and a measure of utility or satisfaction is discussed in Section II-A3 infra. Instead of trying to determine a single likely sentence or sentencing point for each of the four cells, one might feel more comfortable determining a range of sentences for one or more cells, especially where bargaining is over the charge and thus only indirectly over...
culor's threshold of a maximum fair sentence is exceeded. The decreasing rate is due to the general principle of diminishing marginal utility which says the more of a good thing one has, the less satisfaction one gets out of each incremental unit.

The principles of diminishing disutility and utility in the sentencing context can be expressed in terms of equations which translate likely sentences into dissatisfaction or satisfaction units. For the average defendant, the equation would probably take the form $\text{DIS} = A(\text{LS})^B$ where DIS is dissatisfaction units, LS is likely sentence expressed in years or other time units, A is the amount of dissatisfaction received if LS is only one time unit, and B equals a positive exponent less than one to which LS is raised to show the degree of increasing dissatisfaction from additional time units. For the average prosecutor, the equation would take the form $\text{SAT} = A(\text{LS})^B$ where SAT is satisfaction units, and the other symbols have meanings like those in the defendants' equation.

If no better data is available, it could be assumed that the A multiplier in both equations equals 1 and the B exponent in both equations equals .5. This means that for the defendant, dissatisfaction, or more accurately, relative dissatisfaction, is the square root of LS. In more concrete terms, if Table 1A were to reflect the principle of diminishing disutility and show relative dissatisfaction scores, then the cells would be cell $c = 0$, $a = 2$, $b = 2.6$, and $d = 3.2$. Likewise, the prosecutor's payoff matrix would be cell $c = 0$, $a = 1.7$, $b = 2.4$, and $d = 2.8$. To determine more precise values for the A and the B in the above equations requires feeding data for some defendants and some prosecutors into a computerized linear regression analysis which then determines the A and B in each equation that best fits the data.

The sentence. For a discussion of the range approach in determining cell payoffs and conviction probabilities, see Section II-B3(c) supra.

Note that although 10 years is $2\frac{1}{2}$ times 4 years in Table 1A, 3.2 dissatisfaction units is less than 2.5 times 2 dissatisfaction units for the corresponding cells, illustrating the principle of diminishing disutility.

The most meaningful kind of utility data to obtain involves asking the respondents a series of questions designed to determine the relative number of satisfaction or dissatisfaction units associated with a two-year sentence, a four-year sentence, and other time units. The questions for the prosecutor can have the form, "Which choice would you prefer, (1) obtaining a two-year jail sentence for a crime that has a ten-year maximum, or (2) having a lottery ticket that gives you a 90 percent chance to get a $1,000 raise in salary and a 10 percent chance to get a $1,000 reduction in salary?" The next question provides for an 80 percent, 20 percent split on alternative 2, and the next question after that might provide for a 70 percent, 30 percent split until the split is obtained where the respondent says he is indifferent between choice alternatives 1 and 2. We then ask a series of questions in which choice alternative 1 involves a four-year jail sentence. With the answers to these kinds of questions for a group of prosecutors, we can then roughly determine through some simple arithmetic manipulation how many relative satisfaction units a two-year sentence, a four-year
For the sake of simplicity, the rest of this article will work with the likely sentence payoffs shown in Table 1A and Table 1B which are stated in time units rather than the above translated satisfaction units. Doing so is also justifiable partly on the grounds that over short distances there is probably a linear relation, rather than a diminishing rate relation, between sentence years and satisfaction such that the diminishing utility phenomena does not significantly affect the defendant's or prosecutor's optimum strategy or their equilibrium point. Perhaps the bargaining area between defendants and prosecutors generally only covers such short distances. The average distance covered by plea bargaining can be determined by the same questionnaire approach designed to develop more precise satisfaction translation equations.28 Regardless of whether one uses the payoff units shown in Table 1 or the translated payoff units, the subsequent arithmetic manipulation is the same for deriving optimum strategies and equilibrium points.27 In turning now to the subject of optimum strategies or bargaining limits, the role of benefits and costs other than sentence years such as the saving of time, money, and reputation will be discussed.

B. The Bargaining Limits of the Bargainers

In plea bargaining, the defendant is like a buyer seeking as low a price as possible. The prosecutor is like a seller seeking as high a price as possible, although like some sellers he is subjected to statutory maximum price control. In fact, the plea bargaining process is like an old-fashioned marketplace where there are no fixed prices on the products, and the potential buyer and potential seller haggle over the price upon which they should ultimately settle. The buyer knows the maximum price which he is willing to pay, but the buyer tries to convince the seller that his limit is much lower than it is. Likewise, the seller knows the minimum price which he is willing to accept, but the seller tries to

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28 See note 25 supra.

27 As an alternative to using the satisfaction units approach in Table 1, one could try an index number approach whereby the worst cell is assigned a −100, the best cell is assigned a +100, and the other two cells receive numbers between −100 and +100, depending on how close they are viewed to the worst or best cell. Such an approach is highly subjective, and it involves units that are not actually used by defendants or prosecutors in plea bargaining (unlike sentencing time units) or even present at a subconscious level (unlike satisfaction units). If no better measuring units were available, however, that index number approach could produce adequate results. See Part II, supra note 7, at Section IV-C1.
make the buyer think that his limit is much higher than it is. If the buyer-defendant's upper limit and the seller-prosecutor's lower limit can be determined, a more realistic assessment may be made about whether and at what point an equilibrium price will be reached.

1. Determining the Likely Sentences At Varying Conviction Probabilities

To obtain a better understanding of the bargaining limits of the bargainers, it is helpful to convert TABLE 1 into a graph that will indicate the likely sentences at all possible conviction probabilities between 0 and 1.0 rather than just at 0 and at 1.0. Such a graph involves showing the perceived probability of conviction (PC) along the horizontal axis, and the likely sentence (LS) along the two vertical axes as in FIGURE 1. The defendant's trial payoff line can be expressed by the equation \( LS_1 = 0 + 10(\text{PC}) \). The subscript 1 indicates the likely sentence from going to trial as contrasted to the subscript 2 which indicates the likely sentence of a non-negotiated plea. Applying the same logic, the defendant's plead line, i.e. what his likely sentence will be from a guilty plea, can be expressed as the equation \( LS_2 = 4 + (7-4)\text{PC} \), or \( LS_2 = 4 + 3(\text{PC}) \). With the equations for \( LS_1 \) and \( LS_2 \), the defendant's perceived likely sentence may be determined for any PC.

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28 Any straight line can be written in the general form \( Y = A + BX \). A is the value of \( Y \) when \( X \) equals 0; B is the number of units change in \( Y \) per one unit change in \( X \). The \( X \) in this example is \( PC \); the \( Y \) is \( LS_1 \), where the subscript 1 indicates that \( LS_1 \) is the likely sentence from going to trial. When \( PC = 0 \), \( LS_1 \) is at zero, the amount in cell \( c \). Similarly, when \( PC = 1.0 \), \( LS_1 = 10 \), the amount in cell \( d \). Therefore, \( LS_1 = 0 + 10(\text{PC}) \), which may be graphed by connecting the 0 point on the left vertical axis with the 10 on the right vertical axis. See note 23, supra on the use of A and B, rather than a and b, to represent regression coefficients.

29 Instead of connecting the 0 and the 10 by a straight line of the form \( LS = A + B(\text{PC}) \), we could have connected them by a decreasing rate line of the form \( LS = A(\text{PC})B \), where A equals 10, and B is any positive number less than 1, e.g., .5; or by an increasing rate line of the form \( LS = A(\text{PC})B \), where A equals 10, and B is a positive number greater than 1, e.g., 1.5. The same thing could be done with any of the four straight lines shown in FIGURE 1, but there is no data or theoretical reason for suspecting these relations are nonlinear. See note 23 supra. Payoff matrices could show the payoffs when \( PC \) or another contingent probability is .5 as well as 0.0 and 1.0, giving us a payoff matrix with three columns rather than two. Doing so would be useful if the payoffs rise in the middle and then fall, or fall in the middle and then rise. One cannot, however, have a payoff matrix with only one probability column, although (as discussed in Section II-B2(b) infra and in Part II, supra note 7, at Section IV-A1) a payoff matrix with only one row or one decisional alternative is possible. As a minimum, the payoff for a 0.0 probability column (i.e., what would happen if the contingent event would not occur) and a 1.0 probability column (i.e., what would happen if the contingent event would occur) must be determined.

30 Using the same logic as was used in note 28, supra, the likely sentence from a guilty plea is 4 when \( PC = 0 \), and 7 when \( PC = 1.0 \), the amounts which are in cells \( a \) and \( b \). The equation of the plead line is then \( LS_2 = 4 + (7-4)\text{PC} \). Moreover, the graph or slope of the line is established by connecting the 4 on the left vertical axis with the 7 on the right.
example, if the defendant perceives his PC to be .5, he would logically perceive his likely sentence on going to trial to be 5 years or 10 times .5, and he would perceive his likely sentence on pleading guilty to be 5.5 years or 4 + 3(.5). These two predictions could be determined algebraically as above, or geometrically by reading up from the .5 on the horizontal axis of Figure 1 up to the D trial line and then up to the D plead line.

The prosecutor's perceptions of what he thinks the payoffs will be can also be similarly graphed. When so graphed, the P trial line, the dash line in Figure 1, yields a prediction equation of $LS_1 = 0 + 8(\text{PC})$. The P plead line, the X line, yields a prediction equation of $LS_2 = 3 + (6-8)\text{PC}$, or $LS_2 = 3 + 3(\text{PC})$. This means that if the prosecutor perceives the case as being a .4 probability, then the prosecutor will tend
to predict that the defendant will receive a sentence of 8(.4) or 3.2 years if he goes to trial, and 4.2 years if he pleads guilty before a judge.\textsuperscript{81}

2. Determining the Bargaining Limits from the Likely Sentences
   
   (a) Numerical and graphical interpretation
   
   Once the likely sentences that correspond to any conviction probability have been calculated, those likely sentences may be used to determine the bargaining limits for the buyer-defendant and the seller-prosecutor. From the above calculations, the defendant with a PC of .5 would prefer to go to trial rather than plead guilty before a judge since he thinks he will get 5.5 years from a guilty plea, but only an average of 5 years from going to trial. The defendant, however, will be willing to work out a bargain with the prosecutor if the prosecutor will offer him anything less than five years. Likewise, from these calculations the prosecutor with a PC of .4 would prefer to have the defendant plead guilty before a judge rather than go to trial since the prosecutor thinks the defendant will average only 3.2 years by going to trial rather than 4.2 years by pleading guilty. The prosecutor, however, will be willing

\textsuperscript{81} An alternative to the regression approach for relating LS to PC is a weighted average approach. A weighted average approach involves saying that \( LS_i = (1-PC)c + (PC)d \). Thus, if the prosecutor perceives PC to be .4, then by this equation, \( LS_i = (1-.4)0 + (.4)8 = 3.2 \) years. What we are in effect saying is that a good way to determine the likely sentence upon going to trial of a case with a .4 PC is to calculate a weighted average of all the payoffs in which each payoff is weighted by the probability of its occurrence. The formula for a weighted average or mean equals: (1) the sum of the weights times the associated scores divided by (2) the sum of the weights. In this context we substitute the probabilities 1–PC and PC for the weights and c and d for the scores. Since the sum of 1–PC and PC equals 1, the average or the best expectation is simply the sum of \((1-PC)c + (PC)d\). Likewise, the weighted average formula for \( LS_a \) equals \((1-PC)a + (PC)b\), where \( a \) and \( b \) refer to cells a and b rather than to the regression coefficients. If the regression approach is expressed in terms of cell letters, then \( LS_i = c + (d-c)PC \), and \( LS_a = a + (b-a)PC \). The weighted average approach and the regression approach produce identical results because they are algebraically equivalent when they are both simplified by removing the parentheses.

When we use the weighted average approach, we are in effect using the mean of the population or universe of cases like the case at bar (as perceived by the defendant or prosecutor) in order to estimate the likely sentence score of the case at bar. In other words, we are using the mean of the universe to predict the mean of a sample where the sample size is one. The mean provides the best estimation because it tends (in the usual statistics usage) to minimize deviations from the actual score of the sample to the estimated score of the universe, or because it tends (in our decision-theory usage) to minimize deviations from the perceived score of the universe (or set of cases) to the estimated score of the sample (or immediate case). Likewise, when we use the regression line approach, we are in effect using a regression equation for the perceived universe in order to estimate the likely sentence score of the case at bar. The regression line provides the best estimation because it tends (in the statistics usage) to minimize deviations from the actual cases to the estimated regression line, or because it tends (in decision theory) to minimize deviations from the perceived regression line to the case at bar whose likely sentence is being estimated.
to work out a bargain with the defendant if the defendant will accept anything over 3.2 years. In this simple example, the defendant's upper limit or fall-back position is 5 years, and the prosecutor's lower limit or fall-back position is 3.2 years. The defendant has no lower limit except for the fact that he cannot receive a sentence lower than 0. Likewise, the prosecutor has no upper limit except for the fact that he cannot obtain a sentence greater than the statutory or ethical maximum.

What is now needed is a more general method of calculating the defendant's upper limit and the prosecutor's lower limit. A method is required which will apply regardless of how each party numerically perceives the probability of conviction or regardless even of how each perceives the cell payoffs. Figure 1 provides a simple graphic or geometric method for doing that. The defendant's upper limit is along the thickened or bold-faced D trial line until that line intersects with the D plead line and then along the thickened dotted line. The defendant will always select the lower of the two values. At any given conviction probability, the defendant would, of course, accept an offer from the prosecutor that is below that thickened kinked positive concave line because that line represents the best set of likely sentences the defendant can achieve either by going to trial or pleading guilty before a judge. Likewise, the defendant should reject any final offer from the prosecutor that is above that thickened line at any given perceived conviction probability because the defendant can do better than such an offer by either going to trial or pleading guilty before a judge.

By similar reasoning, the prosecutor's lower limit line is the thickened dashed P trial line up to the intersection with the P plead line and then along the thickened P plead line of X's. The prosecutor should be willing to accept any offer the defendant makes above that thickened line because the prosecutor logically reasons that if the defendant wants to minimize his sentence and is as knowledgeable as the prosecutor, then the defendant by going to trial or pleading guilty before a judge can bring the defendant's sentence down to the prosecutor's thickened line level or limit. Likewise, the prosecutor should reject any final offer from the defendant that is below that thickened line at a given perceived conviction probability. The prosecutor should logically reason that such a rejection would force the defendant to either go to trial or plead guilty before a judge, thereby resulting in a higher sentence than the point below the prosecutor's thickened line would mean at that perceived PC.

Figure 1 indicates the upper limit for the defendant and the lower limit of the prosecutor at any conviction probability given each party's
perceived cell payoffs from Table 1. By following the same reasoning used to derive Figure 1 from Table 1, a similar figure or graph for any set of defendant cell payoffs and prosecutor cell payoffs could be derived.\textsuperscript{22} The defendant's upper limit thickened line will always be the combination of his two lines which produces the lowest thickened line that can be made. Likewise, the prosecutor's lower limit thickened line will always be the combination of his two lines that also produces the lowest thickened line.

(b) Limits based on only one alternative to plea bargaining

This scheme presents, in effect, a buyer-defendant haggling with a seller-prosecutor while operating in an oligopolistic market, \textit{i.e.}, a market with only a few sellers. If the buyer-defendant cannot bring the seller-prosecutor down below the buyer-defendant's maximum price level, the defendant can turn to the two other sellers whose anticipated prices determine the defendant's bargaining limit. The other sellers are a judge in an unbargained guilty plea situation and a judge or a jury in a bench or a jury trial.

If the first alternative seller, \textit{i.e.} the judge in a guilty plea situation, is too cooperative with the prosecutor, they in effect have the equivalent of a price conspiracy and both will offer the same sentence. The buyer-defendant then has only one alternative seller from whom he can buy, namely the bench or jury trial or possibly just the jury trial. In the case of the one alternative seller, the defendant's limit line is the same as the defendant's trial line, and likewise the prosecutor's limit line is the same as the prosecutor's trial line.\textsuperscript{33}

In certain cases the opposite situation may exist where the defendant only sees the plead line because going to trial is not a meaningful alternative given the trivial nature of the sentences involved even if the defendant is certain he would be acquitted. This is often true for defendants in traffic violation cases and especially parking meter violation cases. The maximum fine involved in a parking meter violation is so small relative to the cost of either a bench trial or a jury trial that many inno-

\textsuperscript{22}Regardless of what the cell payoffs are, the \textit{Y-intercept} or left vertical axis intersection of (1) the defendant's D trial line will be the payoff in his cell \( c \), (2) the defendant's D plead line will be the payoff in his cell \( a \), (3) the prosecutor's P trial line will be the payoff in his cell \( c \), and (4) the prosecutor's P plead line will be the payoff in his cell \( a \). Likewise, regardless of the cell payoffs, the \textit{slope} of (1) the defendant's D trial line will be his cell \( d \) minus cell \( c \), (2) the defendant's D plead line will be his cell \( b \) minus cell \( a \), (3) the prosecutor's P trial line will be his cell \( d \) minus cell \( c \), and (4) the prosecutor's P plead line will be his cell \( b \) minus cell \( a \).

\textsuperscript{33}The questionnaire or interviewing survey will possibly throw some light on the extent to which pleading guilty before a judge provides a meaningful alternative to plea bargaining with the prosecutor.
cent automobile drivers will simply sign the parking ticket, thereby pleading guilty, even though they know that if they wanted to go to the trouble they could be acquitted.\textsuperscript{54}

There are two other related situations where the buyer-defendant or the seller-prosecutor may have his thickened limit line determined by only one of his two lines rather than both. First, a defendant who has a maximax strategy toward risk is one who will always go for the alternative decision that holds out the possibility of a maximum gain.\textsuperscript{55} This means that if the lowest sentence is in cell $c$ as one would expect it to be, the maximax or go-for-broke defendant will in effect be blind to the D plead line and only see the D trial line. If the prosecutor is aware that the defendant is a maximax strategist, the prosecutor's lower limit line would be just the P trial line. An extreme maximax defendant is one who only sees the D trial line and who is certain he will be acquitted. His maximum limit would be zero, and he is obviously a difficult target for the plea bargaining process. The uncertain maximax defendant can, however, be bargained with since he is not certain that he will be acquitted on trial.

Second, a defendant who has a minimax strategy toward risk is one who will always go for the alternative decision that holds out the possibility of a minimum loss. This means that if cell $b$ is less than cell $d$ as one would expect it to be, the minimax or cautious defendant will

\textsuperscript{54} Where the defendant only sees the pleading line as a possibility, that may be due to the fact that his perceptions of the cell payoffs are such that the adverse payoff in cell $c$ is greater than the adverse payoff in cell $a$, and likewise cell $d$ is greater than cell $b$. In such a situation, the pleading line is said to dominate the trial line as an alternative. In the plea bargaining context, this means at all points the pleading line is below the trial line. Such a situation, however, would only be likely to occur if the defendant were considering goals other than sentence minimization. Where the defendant only sees the trial line as a possibility, this may likewise be due to the fact that the trial line dominates or is always better than the pleading line, as indicated by the trial cells being always higher than the corresponding pleading cells.

\textsuperscript{55} A maximax strategist is literally one seeking to maximize his maximum possible gain, as contrasted to a minimax strategist who is literally one seeking to minimize his maximum possible loss. The maximax strategist will choose the alternative that will give him the biggest gain when the contingent probability is favorable, even though it may also give him the biggest loss when the contingent probability is unfavorable. The minimax strategist will choose the alternative that will give him the smallest loss when the contingent probability is unfavorable, even though it may also give him the smallest gain when the contingent probability is favorable. For further details on these two general types of decision-makers is a context broader and more abstract than plea bargaining, see \textit{Richmond}, \textit{supra} note 8, at 32-38 and 504.
be blind to the D trial line and only see the D plead line. An extreme minimax defendant is one who only sees the D plead line and is certain he will be convicted. His maximum limit would be whatever is in cell $d$, and he is obviously an easy target for the plea bargaining process. The uncertain minimax defendant can, however, break off negotiations if the prosecutor is seeking the maximum cell $d$ sentence since such a defendant is uncertain that he will be convicted.

Most defendants probably do not follow a pure strategy of considering trial as the only alternative to plea bargaining or of considering pleading before a judge as the only alternative. Instead, they recognize both payoff rows and will either go to trial or plead guilty depending on which of those two alternatives will give them the lowest likely sentence at their perceived probability of conviction. They will thus follow a mixed strategy of sometimes going to trial and sometimes pleading guilty as an alternative to unsuccessful plea bargaining depending on what their perception of PC is.

Closely related to the situation where the defendant sees only going to trial or only pleading before a judge as the alternative to plea bargaining is the situation where plea bargaining is not a meaningful alternative. In other words, the defendant-buyer typically has three sellers from which he can buy, the trial court, the pleading judge, or the bargaining prosecutor. In non-typical situations, one or two of these sellers are eliminated as alternatives. The plea bargaining alternative may be eliminated because the defendant, the judge, or the prosecutor rejects plea bargaining under the circumstances. The defendant may reject the idea of having anything to do with plea bargaining when the transaction or settlement costs in terms of time or money, especially if defense counsel has to be hired, are simply too high relative to the incremental benefits that can be obtained by plea bargaining rather than by simply pleading guilty to the maximum penalty. As mentioned earlier, parking violations and other minor traffic matters are instances where defendants rarely resort to plea bargaining. A similar situation exists if the judge refuses to accept a plea bargain negotiated by the prosecutor and defendant. In that situation, the parties either have to renegotiate in light of the constraints imposed by the judge, or they have to consider plea bargaining as an unavailable alternative. Such a situation may be fairly common with regard to bargaining over sentences, although not with regard to bargaining over what the charge should be. The third and

86 If the defendant pleads guilty in reliance on the prosecutor's promise that the judge will give a certain sentence and the judge fails to do so, the defendant normally cannot withdraw his guilty plea. This, however, is usually known to defense counsel and experienced
possibly least common situation is where the prosecutor refuses to have any dealings with the defendant because of the nature of the defendant's crime, the defendant's character, or the custom in that area. In all of the situations where plea bargaining is eliminated as an alternative, the defendant is faced with a decision theory problem where he must decide between going to trial or pleading guilty before a judge. This decision must be based on the defendant's perception of which alternative will produce the lowest likely sentence or the lowest disutility.

(c) Algebraic interpretation

Algebraically, the defendant's bargaining limit can be calculated from the rule: LD, the limit of the defendant, equals the lower of LS₁ or LS₂, where LS₁ is the likely sentence from going to trial, and LS₂ is the likely sentence from pleading guilty before a judge in light of the perceived payoff cells and conviction probability of the defendant. 87 Similarly, the prosecutor's bargaining limit can be calculated from the rule: LP, the limit of the prosecutor, equals the higher of LS₁ or LS₂, where LS₁ is the likely sentence from going to trial, and LS₂ is the likely sentence from pleading guilty before a judge in light of the perceived payoff cells and conviction probability of the prosecutor. 88 Although the defendant and the prosecutor calculate their bargaining limits in a similar manner, the defendant will only accept an offer that is below his bargaining limit, and the prosecutor will only accept an offer that is above his bargaining limit. The conviction probability at which the likely sentence from pleading before a judge becomes lower than the likely sentence from going to trial can be easily determined. To do so for the defendant with the data available involves determining the PC at the point where LS₁ equals LS₂, i.e. where \( PC = .57 \). This means that if the defendant perceives PC to be less than .57, he should defendants, and they will not plead guilty as part of a plea bargain unless they feel almost sure the judge will abide by the prosecutor's recommendation. If the prosecutor or his successor breaks his promise concerning a recommended sentence, then the guilty plea can be withdrawn. Santobello v. New York, 404 U.S. 257 (1971). No matter how intimidating the likely sentence may appear to be, a defendant's plea of guilty as part of a plea bargain will not be considered a coerced confession. Brady v. United States, 397 U.S. 742 (1970). 89

87 As previously described, \( LS_i = c + (d-c)PC \) using the regression approach, or \( LS_i = (1-PC)c + (PC)d \) using the weighted average approach. Likewise, \( LS_p = a + (b-a)PC \) using the regression approach, or \( LS_p = (1-PC)a + (PC)b \) using the weighted average approach.

88 The formulas for calculating \( LS_i \) and \( LS_p \) for the prosecutor are the same as those for the defendant except one inserts the prosecutor's perceptions of the cell payoffs from his payoff matrix and the probability of conviction.

89 Since \( LS = 0 + 10(PC) \) and \( LS = 4 + (7-4)PC \), therefore \( PC^* \) or the intersection PC is found by solving for PC in the equation \( 0 + 10(PC) = 4 + (7-4)PC \). That equation simplifies to \( 10(PC)-3(PC) = 4 \) which means \( 7(PC) = 4 \), or \( PC = 4/7 = .57 \).
calculate his maximum limit by using the LS$_2$ or trial formula. If instead he perceives PC to be greater than .57, he should calculate his maximum limit by using the LS$_2$, or plead formula. If PC is perceived to be exactly .57 (or more precisely .5714), the defendant is indifferent between the trial and plead alternatives. Conviction probabilities, however, are not likely to be perceived that precisely, nor is a defendant likely to be concerned only with sentence minimization.  

A similar algebraic operation may be performed on the prosecutor's payoff data. The result for the data used above is PC = .60. Thus if the prosecutor perceives PC to be less than .60, he would calculate his minimum limit by using his LS$_1$ trial formula. If, on the other hand, he perceives PC to be greater than .60, he would calculate his minimum limit by using his LS$_2$ or plead formula.

Furthermore, at the point where the trial line crosses the plead line (i.e. where LS$_1$ = LS$_2$), then that probability of conviction (PC*) equals \((a-c)/(a-b-c+d)\). Also where LS$_1$ = LS$_2$, then that likely sentence (LS*) equals \((ad-b-c)/(a-b-c+d)\). Applying this form-

40 A perceived or actual probability can only range from 0.0 to 1.0. Even though a probability cannot be greater than 1.00, PC* can be greater than 1.00 if in a given situation, pleading guilty without a bargain were always better or dominant than going to trial, i.e. if cell $a$ involved a lower sentence than cell $c$, and cell $b$ involved a lower sentence than cell $d$. Likewise, although a probability cannot be less than 0, PC* can be less than 0 if in a given situation going to trial were always better or dominant than pleading, i.e. if cell $c$ involved a lower sentence than cell $a$, and cell $d$ involved a lower sentence than cell $b$. In either situation, one would always choose to plead, or always choose to go to trial by in effect rounding PC* down to 1.00 if it is greater than 1.00 or rounding PC* up to 0 if it is less than 0.

41 If LS$_1$ = LS$_2$ then \(0 + 8(\text{PC}) = 3 + (6-3)\text{PC}\). The solution produces PC = .60.

42 This equation may be proven since we know LS$_1$ = $c + (d-c)\text{PC}$, and LS$_2$ = $a + (b-a)\text{PC}$. At PC*, LS$_1$ = LS$_2$. Therefore, PC* may be algebraically determined by solving for PC in the equation \(c + (d-c)\text{PC} = a + (b-a)\text{PC}\). Doing so reveals that PC* = \((a-c)/(a-b-c+d)\). The algebraic solution for PC* is:

1. LS$_1$ = LS$_2$
2. $c + (d-c)\text{PC} = a + (b-a)\text{PC}$
3. $(d-c)\text{PC} - (b-a)\text{PC} = a-c$
4. PC[$(d-c)-(b-a)$] = $a-c$
5. PC* = $(a-c)/(a-b-c+d)$

43 The algebraic solution for LS* is:

1. LS* = $c + (d-c)\text{PC}$
2. LS* = $c + (d-c)[(a-c)/(a-b-c+d)]$
3. LS* = $c + ((ad-ac-cd + c^2)/(a-b-c+d)]$
4. LS* = $[(a-b-c + d)\text{PC} + (ad-ac-cd + c^2)]/(a-b-c+d)$
5. LS* = $(ac-bc-c^2 + cd + ad-ac-cd + c^2)/(a-b-c+d)$
6. LS* = $(ad-bc)/(a-b-c+d)$
ula to the hypothetical defendant, his PC* is .57 probability, and his LS* is 5.71 years. This means that if the defendant's limit (LD) is less than 5.71 years, and the prosecutor will not come down to LD, then the defendant will go to trial. If the defendant's limit is more than 5.71 years at his perceived PC, and the prosecutor will not come down to LD, then the defendant will plead guilty before the judge in order to minimize his sentence.

3. Determining the Conviction Probabilities
   (a) PC in general

   As the previous discussion of Figure 1 indicates, perceived probability of conviction is quite important in determining the defendant's upper bargaining limit and the prosecutor's lower bargaining limit. This is so because those limits differ for every perceived probability of conviction since they are not horizontal lines. They rise positively with increases in PC since the greater the PC the greater the LS. They kink where the trial line intersects the plead line since at that PC point the defendant is better-off pleading guilty than going to trial. Given the importance of PC, it is appropriate to say something about how defense counsel, prosecutors, or social scientists might determine the conviction probabilities for various types of cases.

   The methods available for determining PC are somewhat similar to those discussed previously in regard to determining the likely sentence in each cell of the payoff matrices. The most common method is for a prosecutor or defense attorney to rely on his own unquantified experience. Another method is for the researcher to ask persons knowledgeable in a given area about specific cases or types of cases. Their responses would then be averaged in order to arrive at a consensus PC or guideline PC's. As alternatives to simply averaging the PC or cell payoff estimates of experts, the researcher might (1) bring them together for a group discussion before they answer individually or collectively; (2) prod them to reveal the assumptions behind their estimates in order to recycle those estimates and assumptions to obtain consensus and further clarification; or (3) weight the estimates, if they are diverse, before averaging them, with weights proportionate to (a) subjective ranking or rating of the experts, (b) their own self ratings, or (c) their past predictive accuracy.

   In asking questions about PC's for a given type of case, each respondent can be asked to give a most likely PC, a low but still reason-

44 See Section II-A2 supra.
able PC, and a high but still reasonable PC. If those estimates tend to have a symmetrical distribution, each respondent's high and low estimates may simply be averaged to get his best or mean estimate rather than using his most likely or modal PC, although the mean and mode should be nearly equivalent where symmetry tends to be present. If the high and low estimates are not equidistant from the most likely, or mode, estimate, then the best or mean estimate can be obtained by adding the low estimate plus the high estimate to four times the most likely estimate, and dividing that sum by six. This same approach of getting three estimates for each point can also be used in estimating the payoff cells as well as the conviction probability.\footnote{On the use of survey data to determine outcome probabilities, see Kotler, \textit{supra} note 22; Hüber, \textit{supra} note 22. For a further discussion of the how and why of handling symmetrical estimates, see Richmond, \textit{supra} note 8, at 487-491, 220-224. Three estimates for each point can be meaningfully obtained for a payoff cell by asking for an estimate below the most likely estimate that is likely to occur less than a certain percentage of the time with the given type of case being considered. Likewise, a high estimate may be explained as an estimate above the most likely estimate that is also likely to occur less than the same certain percentage of the time with the given type of case being considered.}

More sophisticated and less obtrusive but harder to apply methods involve gathering data concerning relevant variables for a large sample of cases. One computer card could be used for each case to indicate how the case was positioned on each of the relevant variables and whether the defendant or prosecutor won. Relevant variables might include characteristics of the defendant, victim, witnesses, judge, jury, and lawyers. Characteristics of the evidence such as the presence of eyewitnesses, confessions, fingerprints, ballistics tests and other matters may be more relevant but harder to categorize. The nature of the crime may also bear some relation to the probability of conviction.

These computer cards can then be processed to generate regression or discriminant equations of the form $Y = b_1X_1 + b_2X_2 + \ldots + b_nX_n$, where the b's represent regression or discriminant weights determined by the computer in order to obtain an equation that represents the best fit to the data. The $Y$ indicates the predicted case outcome which can range from 1, meaning defendant is convicted, to 0, meaning defendant is acquitted. The $X$'s represent the scores in the case to be predicted for the characteristics of the personnel, the evidence, and the crime. Applying the above equation to a given case will yield a score that can be treated like a probability, or better yet, for each .10 interval from 0 to 1, the percent of cases in the interval which were lost by the defendant may be determined. That percent can then be used as a more meaningful
indication of the probability of conviction when the interval in which the
case being predicted falls is known. 46

To clarify the terminology, there are four concepts related to con-
viction probabilities that need to be distinguished and emphasized. The
first is $PC$ as perceived by the defendant. The second is $PC$ as perceived
by the prosecutor which may be similar to that perceived by the de-
fendant or by his lawyer if both the prosecutor and defense counsel are
relying on similar information. The third is the true $PC$ or the $PC$ per-
ceived by an omniscient being. It is always 1.0 or 0.0 since an omniscient
being would know with certainty whether the defendant will be convicted
or acquitted. Although the defendant or the prosecutor may sometimes
think they know with certainty that $PC$ is 1.0 or 0.0, they obviously
cannot know with the absolute certainty that an omniscient being could.
The $PC$ of both the defendant and the prosecutor should move closer
to 1.0 or 0.0, depending on whether the defendant will be convicted or
acquitted, as the case gets closer to trial, closer to a verdict, or as the de-
fendant and prosecutor acquire additional information. The fourth con-
cept is not the probability of the defendant being convicted, but rather
the probability of whether or not he is actually guilty. The prosecutor in
seeking a plea bargain generally thinks the defendant is guilty although
the prosecutor may recognize that the probability of getting a conviction
is substantially less than 1.0, especially where incriminating but non-
admissible evidence is involved. The defendant by plea bargaining is not
necessarily admitting his guilt, and in some cases he may strike a plea
bargain when he believes he is innocent, but feels that the probability

46 For a discussion of various methods designed to obtain case outcome probabilities
from case data, see Nagel, Judicial Prediction and Analysis from Empirical Probability
Tables, 41 Ind. L.J. 403 (1966). Those methods include regression analysis, discriminant
analysis, Bayesian probability, and the Sonquist-Morgan automatic interaction detector.

Using the same nationwide sample of 11,256 criminal cases mentioned in note 21 supra,
an average probability conviction for each major crime may be obtained by simply
observing, for any given crime, how many of the cases involving that crime resulted in a
conviction on trial. For example, looking at the murder cases indicates that of the 66
murder cases that went to trial for which information was available, 70 resulted in guilty
verdicts and 16 resulted in an acquittal verdict. This means that the empirical probability
of conviction in that nationwide sample of murder cases was .81. With the same data,
a probability may be deduced by trying to predict whether or not the defendant will be
convicted from various characteristics of his case plus the crime with which he has been
charged. Thus, if the crime is known to be murder and the defendant is known to be
indigent, by using the Bayes method of determining probabilities, it can be said that the
conviction probability in light of the data and those two circumstances is .84. This esti-
imated $PC$ is derived from the facts that (1) .81 of the murder trial cases result in con-
victions and .19 in acquittals; (2) about .31 of those 70 in the sample convicted of murder
in a trial were indigent enough to have court-appointed counsel; (3) about .25 of those
16 in the sample acquitted of murder were indigent; and (4) .84 = (.81 * .31) /
[(.19 * .25) + (.81 * .31)]. For further details on Bayesian empirical probabilities, see
Richmond, supra note 8, at 145-52, 541-50.
of conviction is too high to risk going to trial. These four concepts can be symbolized PCD, PCP, PC', and PG respectively, although they are usually referred to verbally rather than symbolically elsewhere in this paper.

Knowing what the probability of conviction is between 0 and 1.0 is essential in determining the defendant's upper limit or the prosecutor's lower limit in Figure I. The following sections deal with the problems raised when PC is known to be either 0 or 1.0, (2) PC is known to be within a range between one probability and another, or (3) PC is totally unknown.

(b) PC under certainty of conviction or acquittal

The certainty situation is relatively easy. Since PC is either 0 or 1.0, this situation involves working with only the original payoff matrices of Table 1 rather than the strategies graph of Figure I. If the defendant is certain that he will be acquitted, i.e. that PC is 0, he should reject any prosecutor offer other than an offer to dismiss the charges. Even an offer of probation or a suspended sentence would be too high. Likewise, if the prosecutor is certain the defendant will be acquitted and the defendant knows it, the prosecutor should dismiss the charges rather than seek to plea bargain. Even if the prosecutor thinks, in spite of the lack of convincing evidence, that the defendant is guilty, he should dismiss the charges, unless the prosecutor knows that the defendant is unaware that PC is 0.

If the defendant is certain that he will be convicted, i.e. that PC is 1.0, he should accept any prosecutor offer of a sentence less than his cell d perceived maximum. In order to save his time and money, he should even accept an offer from the prosecutor as high as the cell d maximum if he is truly convinced that PC equals 1.0. Likewise, the prosecutor can hold out for his cell d maximum if he thinks PC is 1.0 although later a prosecutor's discount factor will be discussed whereby the prosecutor may be willing to discount the likely sentence in order to save time and money.47 There are, however, few, if any, realistic situations where either the defendant or the prosecutor is absolutely certain that the defendant will be acquitted or convicted if the case goes to trial.

47 The statements from the text concerning defendants and prosecutors operating under conditions of certainty assume that both the trial line and the plead line are being used. See the discussion of the certainty part of Table 2 in Part II, supra note 7, at Section III-A2 and Section II-B2(b), supra, for the highly unusual defendant who is certain he will be acquitted but still pleads guilty, or who is certain he will be convicted but still goes to trial.
(c) PC within a range between 1.0 and 0.0

The second problem situation regarding the PC perceptions of the defendant and the prosecutor is the situation where one of them does not feel confident that the conviction probability is at a given point. The litigant instead feels that it is within a range between two points. The defendant’s bargaining limit under those circumstances depends on his degree of optimism-pessimism. For example, if the defendant perceives his probability of conviction as being between .2 and .5, he would act as if PC were .2 if he were optimistic. On the other hand, he would act as if PC were .5 if he were more pessimistic. If he is neither an optimist nor a pessimist, but rather has a middling attitude toward his probability of conviction, then he would probably act as if his PC were 3.5 or \((.5 + .2)/2\). On the other side, if we had an optimistic prosecutor who perceived PC as being between .2 and .5, then he would act as if PC were more like .5 and .2 since the optimistic prosecutor perceives PC as being high when the optimistic defendant perceives PC as being low. Likewise, the pessimistic prosecutor in this hypothetical situation would act as if PC were .2, and the middling prosecutor would act as if PC were 3.5.

The notion of optimism-pessimism can be applied to the payoff cells in Table 1 as well as to the conviction probabilities. Thus, a defendant or defense counsel may be quite unsure that cell a involves a likely sentence of 4 years, but may feel reasonably confident that it involves a sentence somewhere between 3 years and 6 years. If such a defendant is optimistic he will act as if that cell means a likely sentence of 3 years, while he might act as if that cell means a likely sentence of 6 years if he is a pessimistic defendant. The middling defendant would assume a sentence of about 4.5 years. Likewise, an optimistic prosecutor would tend to assume the maximum sentence within his perceived range, and a pessimistic prosecutor would tend to assume the minimum sentence within that range.

To determine an individual’s optimism-pessimism coefficient, one could refer first to Table 1A with its a, b, c, and d cells of 4, 7, 0, and 10 respectively. Now determine how cell a would have to be changed, upward or downward, in order to make going to trial as appealing as pleading guilty before a judge or vice versa. For example, a defendant may decide that going to trial is as appealing as pleading guilty before a judge when the cells are changed to 2, 7, 0, and 10, respectively. Given this data, the optimism-pessimism coefficient may be calculated, What the individual defendant has in effect said is that when the cells
are 2, 7, 0, and 10, then \( L_S_1 \) will be equal to \( L_S_2 \). This is the same thing as saying that \( 0 + (10-0)PC = 2 + (7-2)PC \). Thus, solving for \( PC \) in that equation, the measure of his degree of optimism-pessimism is determined. This yields a \( PC \) or an \( 0-P \) coefficient of \( 2/5 \) or .40 which is fairly optimistic as compared to .50, but not as optimistic as .30.\(^{48}\)

After determining the defendant's optimism-pessimism coefficient through the above method, the coefficient can be applied to finding a point within the estimated range of a payoff cell or a conviction probability. For example, if the perceived range in a payoff cell of the defendant is 3 to 6 and the \( 0-P \) coefficient is .5, then the difference between 3 and 6 should be split, yielding the point 4.5. If the \( 0-P \) coefficient is .6, then take .6 of the difference, \( i.e. \) 6 times 3, which equals 1.8. This yields a working value or point estimate for the defendant's cell of 4.8 (or 3 + 1.8). A similar approach can be used to find a working value for \( PC \) between the range of .2 and .5, \( i.e. \) multiply the .3 difference between .2 and .5 by the \( 0-P \) coefficient, and add that product to .2. A similar procedure can be followed to reduce the prosecutor's perception of \( PC \) from a range to a point. The only difference is that when the range is multiplied by the \( 0-P \) coefficient, the product is subtracted from the top of the range for the prosecutor, whereas it is added to the bottom of the range to obtain a meaningful point for the defendant.\(^{49}\)

\(^{48}\)A defendant's optimism coefficient is equal to \( PC^* \) when the cell payoffs are adjusted so that the expected value or likely sentence of going to trial is perceived as about equal to the likely sentence of pleading guilty before a judge. This method for calculating an optimism-pessimism coefficient or a \( PC^* \) value designed to reflect one's optimism-pessimism was developed by Leonid Hurwicz. See Richmond, supra note 8, at 33-34.

\(^{49}\)An alternative approach to handling the problem of the \( PC \) range is to think in terms of a vertical probability band in Figure 1 rather than a probability point. For example, a defendant with a perceived \( PC \) range between .2 and .5 in Figure 1 would have a limit between 2 years and 5 years. This would mean that the prosecutor would have to make an offer below 2 years is order for the defendant to accept it if the defendant only accepts offers below his limit. Thus, the probability band approach would arrive at the same result as the optimistic defendant concept would. Likewise, the probability band approach applied to the prosecutor would produce the same result as the optimistic prosecutor concept. Since not all defendants and prosecutors are optimistic, this probability band approach is less likely to reflect empirical reality than thinking in terms of three types of defendants and three types of prosecutors on an optimism-pessimism scale.

If the defendant's cell \( a \) in Table 1 ranged from 3 to 6 years instead of being exactly 4, a limit band approach applied to Figure 1 would result in an elongated triangle lying on its side with the base extending from the 3 to the 6 on the left vertical axis over to the apex 7 on the right vertical axis. This elongated triangle would constitute the defendant's new D plead band. The defendant, under these circumstances, would reject all offers by the prosecutor which are not below a line from 5 years to 7 years when the defendant's \( PC \) is between .5 and 1.0 instead of the higher former D plead line in Figure 1. The limit band approach has the effect of generating a limit for the defendant which is the same limit as the one for the optimistic defendant who perceives the likely sentence of cell \( a \) to be between 3 and 6 and who therefore has a D plead line extending from 3 when \( PC = 0 \), to 7 when \( PC = 1.0 \). As contrasted to the more realistic approach of using the three-point optimism-pessimism scale, the limit band approach is as inapplicable as the probability
(d) PC totally unknown

The third problem situation of a totally unknown PC presents the extreme version of the defendant or the prosecutor who thinks of PC in terms of a range. In this situation, the defendant or the prosecutor thinks PC could just as easily be 0 or 1.0. The point on that complete range at which the defendant or the prosecutor will operate depends on his degree of optimism-pessimism. If the defendant is highly optimistic, he will act on the assumption that PC is 0 or close to 0. If he is highly pessimistic, he will act on the assumption that PC is 1.0 or close to 1.0. Likewise, if the prosecutor is highly optimistic, he will act as if PC is about 1.0, and if he is highly pessimistic, he will act as if PC is about 0. If, however, either the defendant or the prosecutor has a middling degree of optimism, he will act as if PC is about .5. In the hypothetical case previously discussed, such a defendant would have an upper bargaining limit of 5 years, since Figure 1 and the algebraic formulas indicate that a bargaining limit of 5 years corresponds to a PC of .5.

A defendant who is unknowledgeable as to PC and middling on optimism-pessimism might flip a coin to determine whether he prefers to go to trial or to plead guilty before a judge. If he flips a coin between trial and pleading, the defendant is saying that he is indifferent in terms of his perceptions and values between trial or pleading guilty before a judge. Such a defendant is acting as if PC were PC*. In other words, he is acting as if the likely sentence from going to trial is the same as the likely sentence from pleading guilty before a judge. In the hypothetical example, such a defendant would be acting as if PC were .57 rather than acting as if PC were .5. His limit would thus be 5.7 years rather than 5 years since he could fall back on an expected 5.7 year sentence by going to trial or pleading guilty before a judge if negotiations with the prosecutor break down.

The better of these two alternatives for our unknowledgeable defendant with middling optimism-pessimism is that which assumes PC equals .5, rather than flipping a coin which impliedly assumes PC equals .57. It is the better alternative because we are trying to capture what is really likely to happen in the rare situation of a defendant who is completely unknowledgeable as to PC. It is also a simple method that pro-

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provides an estimation of PC with a minimum of effort, whereas assuming PC equals .57 involves a somewhat complicated chain of thinking which runs from (1) flipping a coin between going to trial and pleading guilty before a judge to (2) thinking that doing so means LS_1 equals LS_2 and to (3) thinking this means PC equals .57. Therefore, to avoid this complicated chain of thinking, the PC equals .5 approach will be used elsewhere in this paper.\(^6^0\)

To clarify further the specific terminology, it should be pointed out that the optimistic defendant is not necessarily the same as the maximax defendant. The optimistic defendant is one who sees PC or a cell payoff as being at the lower end of the range of realistic possibilities. The maxi-

\(^6^0\)On the other hand, assuming that PC equals .57 comes closer to satisfying the technical criterion which is used in formal decision theory to make decisions when the contingent probabilities are totally unknown. That technical criterion says to mix decisions between the alternatives of trial and pleading in such proportions as to equalize (1) the average return which will be received when the contingency, i.e., being convicted, does not occur and (2) the average return which will be received when the contingency does occur.

Applying this technical criterion to the hypothetical data in TABLE 1A and FIGURE 1, it is found that if the defendant operates on the assumption that PC equals .5, then 0 percent of the time he will plead guilty, and 100 percent of the time he will go to trial given that data and those alternatives. This means that when he would have been acquitted, his average sentence will be 0 times 4 plus 1 times 0, or the proportion allocated to pleading (which is 0) times cell a (which is 4) plus the proportion allocated to trial times cell c. When he would have been convicted, his average sentence will be 0 times 7 plus 1 times 10. The first sum equals 0, and the second sum equals 10 for a difference of 10 years. On the other hand, if the defendant operates on the assumption that PC equals .57, then 50 percent of the time he will plead guilty, and 50 percent of the time he will go to trial. When he would have been acquitted, his average sentence will then be .5 times 4 plus .5 times 0 or 2 years. When he would have been convicted, his average sentence will be .5 times 7 plus .5 times 10, or 8\(\frac{1}{2}\) years. The difference between 8\(\frac{1}{2}\) years and 2 years under the .57 assumption is smaller than the difference of 10 years under the .50 assumption.

Actually, the smallest difference which could be created between those two averages with the data given would involve pleading guilty before a judge 100 percent of the time and never going to trial as if PC were 1.0. Then the difference between those two averages is 7 minus 4 years, or just 3 years. This same result would occur if the unknowledgeable defendant acted in the most pessimistic way possible by assuming that his conviction probability equaled 1.0. Using the technical criterion, this would be the best thing for the defendant to do. This, however, illustrates how conservative or pessimistic and probably unrealistic that technical criterion or minimax criterion is.

If the minimax or other criterion had indicated that the best strategy when operating under ignorance is to go to trial 65 percent of the time and plead guilty 35 percent of the time, then what would one do if he were involved in only a single case? At a superficial level, it might be said to go to trial since that is what one is supposed to do most of the time. At a more sophisticated level which in the long run will lead to more satisfaction, the answer is to draw the first two-digit number from a random numbers table. If the two-digit number is less than 65, go to trial; whereas if the two-digit number is more than 65, plead guilty before a judge. In game theory, such a randomized strategy is often the optimum strategy on the assumption that the player being advised is totally ignorant of the other player's likely moves. Seldom if ever though will a real-world decision-maker admit to such ignorance that he has to draw random numbers to make important litigation or business decisions. For further discussion of decision making when one is ignorant of the contingent probabilities, see RICHMOND, supra note 8, at 32-38, 535-538.
max defendant is the one who seeks the alternative between going to trial or pleading guilty before a judge that can lead to the best possible payoff regardless of the probability of conviction. Most maximax defendants are also likely to be optimistic defendants, although most optimistic defendants are not necessarily also maximax defendants. Likewise, most minimax defendants are also likely to be pessimistic defendants, although most pessimistic defendants are not necessarily also minimax defendants. Regardless of the correlation between strategy toward the alternatives and optimism-pessimism, it is helpful to keep these concepts separate in order to allow the model to recognize more types of defendants and prosecutors. The separate concepts of risk preferrer and risk avoider will also be introduced later to refer to defendants and prosecutors who enjoy or abhor risk as a non-sentence goal or anti-goal separate in itself, from the basic goal of sentence minimization for the defendant and sentence maximization for the prosecutor.

4. Determining the Bargaining Limits Where Non-Sentence Goals Are Involved

(a) What the other goals are

Thus far, only the defendant's goal of sentence minimization and the prosecutor's goal of sentence maximization, within the constraints of the law and the prosecutor's notions of fairness, have been discussed. Other goals, however, have been mentioned, such as when the certainty probability was discussed in Section II-B3(b) supra. Now is an appropriate time to discuss the other goals since they particularly influence the bargaining limits of the parties (LD and LP) rather than the likely sentence payoff cells: a, b, c, and d. Although it is generally more difficult to quantify goals other than sentence minimization and sentence maximization, it is possible to identify some of the goals and to determine how they tend to influence the limits of the defendant-buyer and the prosecutor-seller.

Other goals of the defendant might include (1) getting out of jail as quickly as possible if he is unable to make bond pending trial, (2) saving the costs of hiring an attorney, or of the additional attorney fees required in going to trial without a court-appointed lawyer, (3) saving the time involved in preparing his case and appearing in court, (4) saving his reputation from the bad publicity often associated with a contested trial even if he is acquitted, and (5) saving himself the anxieties associated with prolonging the outcome of the case. The above five goals are all likely to cause the defendant to accept a higher bargain-
ing limit, thereby increasing the likelihood of the pretrial settlement. He may have still other goals that push him in the opposite direction, thereby making it more difficult for the prosecutor to deal with him. These goals include (1) seeking to delay the outcome of the case in order to give himself prolonged freedom if he is likely to be convicted, (2) seeking the publicity of a trial in those cases where publicity is desired, (3) seeking delay in hopes that the prosecution's case will weaken through the increased forgetfulness and unavailability of witnesses, and (4) seeking the safeguards for the innocent, which also benefit the guilty, that are only associated with trial, such as requiring conviction by a unanimous twelve-person jury.

The main goals of the prosecutor other than sentence maximization include (1) saving the costs of preparing for a trial and appearing in court, (2) reducing the backlog of cases awaiting trial, thus reducing court congestion and delay, (3) increasing the percentage of convictions, and (4) obtaining cooperation from the defendant as a witness or informer in other cases. These four goals encourage the prosecutor to decrease his bargaining limit, thereby tending to avoid trial. Other non-sentence goals, however, might have a partially offsetting effect on these four goals. Such other and opposite goals include (1) seeking the publicity of a trial where the prosecutor may be politically motivated, or (2) seeking the publicity of a trial in order to use the defendant as an example to others even though the likely sentence from the trial might be less than what the prosecutor could achieve through plea bargaining.

A non-sentence goal is not involved if the defendant wants to avoid trial because he thinks a trial will get him a longer sentence or wants to go to trial because he thinks a trial will get him a shorter sentence. That kind of goal is included in the Table 1 payoff matrix and the Figure I bargaining limits. Likewise, the table and figure do take into consideration the prosecutor who avoids trial because he thinks he can get as long a sentence through plea bargaining. This concept

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51 Prosecutors want to maximize the percentage of convictions as well as the sentences received by convicted defendants. To take into consideration that non-sentence goal of maximizing conviction percentages, prosecutors are probably willing to allow an extra discount when PC falls below .50. In fact, that portion of the discount may go up at a roughly linear rate as PC goes down from 1.00 to zero.

Defendants, however, who want to minimize their conviction probabilities will not plead guilty as a bonus since a guilty plea is a conviction. Through the bargaining process, a prosecutor can minimize the number of low PC cases that go to trial. Likewise, through the bargaining process, a defense counsel who has many cases can minimize the number of high PC cases that go to trial. The defendant with one case, however, can only minimize PC by getting better witnesses or evidence on his behalf, not through bargaining whereby he agrees to plead guilty to a reduced charge or recommended sentence.
of "other goals" includes only those goals which do not relate to either party's perception of either the conviction probability or the likely sentences of the payoff cells.

(b) Figuring other goals and their determinants into the calculations

In light of the analogy of the defendant to a buyer and the prosecutor to a seller, it can be said that the defendant-buyer is willing to add a bonus to his price or bargaining limit in order to take his other goals into consideration, assuming, as is generally the case, that his other goals tend more to raise his willingness to pay than to lower it. If they tend to lower his willingness to pay, then he adds a negative bonus. Likewise, the prosecutor-seller is willing to subtract a discount from his price or bargaining limit in order to take his other goals into consideration, assuming, as is generally the case, that his other goals tend more to lower the price he demands than to raise it.

In more quantitative terms, it can be said that the defendant's adjusted limit, i.e. his bargaining limit adjusted for his non-sentence goals, equals LD plus XD where LD is his unadjusted limit and XD is his bonus factor. For example, if the defendant, in accordance with Figure 1, which is based on Table 1, has a 5 year limit at his PC of .5, and he is willing to provide a 10 percent bonus, then his adjusted limit is 5.5 years, and his bonus factor is .5 years. This is comparable to a municipality giving a 10 percent bonus for early construction of a needed bridge. The defendant-buyer is, in effect, seeking early delivery of his purchase or early pretrial resolution of his case.\(^\text{62}\)

The prosecutor's adjusted limit equals LP plus XP where LP is his unadjusted limit and XP is his discount factor. For example, if the prosecutor, in accordance with Table 1 and Figure 1, has a 3.2 year limit at his PC of .4 and he is willing to provide a 15 percent discount, then his adjusted limit is 2.7 years, and his discount factor is a little less than .5 years. This is comparable to a business firm giving a 15 percent discount.

\(^\text{62}\) In terms of the graph shown in Figure 1, adding a 10 percent defendant's bonus so as to avoid the defendant's litigation costs over his settlement costs has the effect of raising the defendant's limit line by 10 percent. Without the bonus, LD, the defendant's unadjusted limit, equals \(c + (d-c)PC\) below \(LS^*\), or \(0 + (10-0)PC\) below 5.7; and equals \(a + (b-a)PC\) above \(LS^*\), or \(4 + (7-4)PC\) above 5.7. With the bonus, the kinked LD line, the defendant's limit line, equals LD plus 10 percent of LD. That new kinked line would run parallel and above the old LD line, and it is symbolized ALD or adjusted limit of the defendant. In terms of Figure 2, see Part II, supra note 7, at Section III-B1, the dashed ALD line is 10 percent above the unshown LD line, so long as the bonus factor remains at 10 percent for the defendant throughout the plea bargaining time-points, although it can change.
cent discount for early payment on an invoice. The prosecutor-seller is in effect seeking early payment on his sale or early pretrial resolution of his case.

Both the defendant's bonus factor and the prosecutor's discount factor are probably closely related to the severity of the case although in opposite directions. Other general factors that help explain the level of XD and XP are the attitudes of the defendant and the prosecutor toward risk. If the defendant is a risk avoider, irrespective of his perception of the payoff cells and his conviction probability, he is likely to be willing to give a higher bonus than he would if he were a risk preferrer. Likewise, if the prosecutor is a risk avoider, meaning he has

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63 In terms of the graph shown in Figure 1, deducting a 15 percent prosecutor's bonus to avoid the prosecutor's litigation costs over his settlement costs has the effect of lowering the prosecutor's limit line by 15 percent. Without the discount, LP, the prosecutor's unadjusted limit, equals $c + (d-c)PC$ below LS*, or $0 + (8-0)PC$ below 4.8; and equals $a + (b-a)PC$ above LS*, or $3 + (6-3)PC$ above 4.8. With the bonus, the kinked LP line, the prosecutor's limit line, equals LP minus 15 percent of LP. That new kinked line would run parallel and below the old LP line, and it is symbolized ALP or adjusted limit of the prosecutor. In terms of Figure 2, see Part II, supra note 7, at Section III-B1, the dotted ALP line is 15 percent below the unshown LP line, so long as the discount factor remains at 15 percent for the prosecutor throughout the plea bargaining time points, although it can change.

64 References to a percentage bonus for the defendant to cover his non-sentence goals and a percentage discount for the prosecutor to cover his non-sentence goals could be avoided if sentences and non-sentence goals could be translated into a common unit of measurement like dollars or satisfaction units. This is in effect what is done in G. Tullock, The Logic of the Law 176-186 (1971); Landes, An Economic Analysis of the Courts, 14 J.L. & Econ., 61-107 (1971). [Hereinafter cited as Landes]. They, however, only work with algebraic symbols. In the real world, it may be virtually impossible to translate all the goals into dollars and especially into satisfaction units, but it may not be so difficult to deal with sentence years and a percentage bonus or a percentage discount.

65 The more severe a case is, the less willing a defendant might be to plea bargain and thus plead guilty, especially when he is an innocent defendant or a defendant with a low perceived conviction probability. Thus, a higher percentage of plea bargains and guilty pleas probably exist in misdemeanor violations than in murder cases. The severity of the case as perceived by the defendant can be measured by looking to the value in the defendant's payoff cell d. It can then be said that $XD = A - B(d)$, assuming a negative linear regression relation between XD and d. The A is the value of XD when cell d is zero, and the B indicates how many units XD changes when d changes one unit. Perhaps through appropriate interviews and questionnaires of defense attorneys, some data could be obtained to establish numerical values for A and B.

The more severe a case is, the more willing a prosecutor might be to plea bargain, since the more severe cases involve more time consumption and other costs which the prosecutor is seeking to avoid. Therefore, his XP regression equation might take the form $XP = A + B(d)$, assuming a positive linear regression relation between XP and cell d. Alternatively, prosecutors might prefer to take big cases to trial because of the possibly favorable publicity from obtaining a conviction in them and because of an unwillingness to be blamed for having reduced the charge. Mather, Some Determinants of the Method of Case Disposition: Decision-Making by Public Defenders in Los Angeles, 8 Law & Society Rev. 187-216 (1974). Perhaps interviews and questionnaires of prosecutors could establish numerical values for A and B, and, perhaps, such data would indicate the degree to which XD differs from XP and why.
risk aversion as one of his additional goals, he is likely to be willing to give a higher discount than he would if he were a risk preferrer.

Still another general variable that shapes the defendant's bonus factor and the prosecutor's discount is the amount of resources available to both sides. If the defendant is rich, the costs of hiring an attorney will mean less to him than if he is more of a lower middle class defendant. Likewise, if the prosecutor or the public defender has abundant resources, he will not be so concerned about the costs of preparing for a trial and appearing in court. Some models that have been developed to explain certain aspects of plea bargaining, especially those dealing with the behavior of prosecutors, include the litigant's resources as an important component. The resources available to each side can also affect the probability of conviction regardless of how guilty or innocent the defendant may actually be. Given the possible tendency of some judges to favor those defendants who contribute more to the gross national product, the resources of the defendant may enable him to obtain a shorter sentence if convicted than other defendants. Still, a high income defendant may suffer more disutility stigma and economic loss from a shorter sentence than a lower income defendant would.


67 A perspective that concentrates on the prosecutor's resources may lead one into an alternative model that involves explaining the prosecutor's behavior in terms of his trying to find an optimum mix of his resources among his cases in terms of their respective probabilities of conviction and their sentences if conviction occurs. That type of behavior is analogous to an investment company manager trying to develop an optimum portfolio of stocks for a client given: (1) the client's resources; (2) the probabilities of certain contingent events that can result in stock increases or decreases; and (3) the amount of increase or decrease if the contingencies occur. W. Baumol, Portfolio Theory: The Selection of Asset Combinations (1974). Michael Fried, in a forthcoming expansion of his paper is experimenting with the application of portfolio analysis to the behavior of prosecutors using empirical data from Detroit, Michigan. See supra note 16. The situation is relatively simple if the prosecutor has only two cases between which his resources must be allocated, and the only defense attorney is the public defender who has the same two cases. The problem then becomes one of developing an indifference curve for the prosecutor showing what combinations of resources between those two cases would provide him with equal satisfaction at a given level of satisfaction. A similar set of indifference curves could then could be developed for the public defender, and both sets of indifference curves could be placed in an Edgeworth box format roughly related to that described in Part II, supra note 7, at Section III-A1(c). That approach would enable a determination of whether the bargainers would be likely to arrive at a settlement and within what range such a settlement might occur. E. Mansfield, Microeconomics: Theory and Applications 20-49 (1970); Birmingham, Damage Measures and Economic Rationality: The Geometry of Contract Law, 1969 Duke L.J. 49-71; and Lachman, supra note 56. The situation, however, becomes quite complicated if the number of cases is increased beyond two, the number of defense attorneys increased beyond one, and probabilistic or stochastic considerations are added to this non-probabilistic or deterministic model. Portfolio analysis might be especially applicable to a personal injury plaintiff or defense lawyer trying to decide how to allocate his scarce resources among a set of cases, each one of which has...
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By way of algebraic summary, it can be said that where ALD is the adjusted limit of the defendant, \( ALD = LD + XD \). If XD can also be expressed as a percentage \( \%XD \), then we can say \( ALD = LD + (\%XD \cdot LD) \). This equation can also be written as \( ALD = LD(1 + \%XD) \) with \( \%XD \) expressed as a decimal. Similarly, it can be stated for the prosecutor that, where ALP is the prosecutor's adjusted limit, \( ALP = LP - XP \), or \( ALP = LP(1 - \%X) \). For the sake of simplicity, however, only the terms LD and LP will be used in the further portions of this article dealing with the determination of the equilibrium point toward which the defendant and prosecutor tend to move. The equilibrium models presented, however, would apply equally well even if the ALD and ALP approach were used.

At the simplest level, the defendant is seeking to minimize his sentence subject to the constraint that the sentence cannot be a value less than 0. The prosecutor in that context is seeking to maximize the defendant's sentence subject to the constraint that the sentence cannot be greater than the maximum provided for in the statutes or the maximum which the prosecutor ethically considers appropriate to the circumstances of the case. At a more complete level of analysis, the defendant is seeking to minimize both his sentence and his non-sentence costs and to maximize his non-sentence benefits. Likewise, the prosecutor is seeking to maximize the defendant's sentence and also the prosecutor's non-sentence benefits and to minimize his non-sentence costs. These non-sentence goals are taken into consideration in calculating the adjusted limit of the defendant and the adjusted limit of the prosecutor. At a still more complete level of analysis, both the defendant and the prosecutor are seeking to maximize their respective satisfaction units. That concept requires arithmetically transforming both the sentence payoff cells and the non-sentence considerations,\(^{68}\) so as to reflect the principles of diminishing incremental satisfaction and dissatisfaction. Fortunately for the sake of simplicity, such a transformation is probably unnecessary over the short ranges that are the subject of plea bargaining. The next analysis must be of when and how that range is narrowed to a settlement point.

Note

This article will be completed in the Fall, 1976, issue of the *Indiana Law Journal*. The completion includes further material re-

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\(^{68}\) See Section II-A3 *supra.*
lating to (1) the results of clashes between different types of defendants and prosecutors, (2) the dynamics of the process of converging toward an equilibrium from the initial offers and counter offers, (3) analogies to out of court civil settlements, (4) practitioner and policy implications, and (5) future research applications that build on the basic notions presented regarding plea bargaining, decision theory, and equilibrium models.