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Plea Bargaining, Decision Theory, and Equilibrium Models: Part II

STUART S. NAGEL* & MARIAN NEEF**

The following material represents the completion of the article begun in the Summer, 1976, issue of the *Indiana Law Journal*. Appendices listing terms and formulas used are included at the end of this article.

The first part of this article included material concerning (1) how defendants and prosecutors perceive the probability of a conviction and the sentence that will be received from a conviction, (2) how defendants and prosecutors implicitly use that information in order to determine their respective bargaining limits, and (3) how those bargaining limits are adjusted for non-sentence goals.¹

The model views the plea bargaining process as analogous to a buying/selling transaction in a market that has no fixed prices, much like that of a push-cart peddler. The defense counsel or defendant is the buyer seeking as low a price, charge, or sentence as possible. The prosecutor is a seller seeking as high a price, charge, or sentence as possible within the constraints imposed by the criminal statute and possibly his sense of equity. Each has in mind a rough notion of how high or low he is willing to go before breaking off negotiations and turning to the trial alternative.

How high the defendant-buyer is willing to go depends on his perception of the probability of his being convicted and the sentence he is likely to receive if he is convicted. How low the prosecutor-seller is willing to go also depends on his perception of the conviction probability and the likely sentence. By multiplying each party's perception of the conviction

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¹Nagel & Neef, *Plea Bargaining, Decision Theory, and Equilibrium Models: Part I*, 52 IND. L.J. 987 (1976) [hereinafter referred to as *Part I*]. For a short summary of *Parts I* and *II*, see Nagel & Neef, *The Impact of Plea Bargaining on the Judicial Process*, 60 A.B.A.J. 1020 (1976).

probability times the likely sentence, one can obtain the expected value of going to trial for either. Those expected values represent the upper bargaining limit of the defendant and the lower bargaining limit of the prosecutor before adjustments are made for other considerations. They are the outside limits in the sense that if the other side will not go that far that limit is the expected value that can be achieved by turning to the trial alternative.

The defendant's upper limit needs to be adjusted for such non-sentence goals as getting out of jail while awaiting trial, avoiding the cost of hiring an attorney, and waiving the due process safeguards associated with a jury trial. Those non-sentence goals generally result in the defendant-buyer's being willing to offer a bonus above his base price or unadjusted limit for early delivery of the product or resolution of the case. The prosecutor's lower limit needs to be adjusted for such non-sentence goals as conserving his litigation resources, preserving his high conviction percentage, and waiving the use of the defendant as an example to others. Those non-sentence goals generally result in the prosecutor-seller's being willing to offer a discount below his base price or unadjusted limit for early payment on the invoice or resolution of the case.

Where the defendant-buyer has a choice of (1) going to trial, (2) pleading guilty without a bargain, or (3) negotiating a plea, his upper limit in dealing with the prosecutor is likely to correspond to the lower of the two sentences that he perceives as being likely from the first two alternatives. Likewise, where the prosecutor-seller recognizes that the defendant has the above three choices, his lower limit in dealing with the defendant is also likely to correspond to the lower of the two sentences that he perceives as being likely from those two alternatives available to a negotiated plea.

III. EQUILIBRIUM MODELS APPLIED TO THE DEFENDANT AND THE PROSECUTOR

A. *Results of Clashes Between Different Bargainers*

Since a high percentage of, but not all, negotiations between defendants and prosecutors result in out-of-court settlements, a useful model should be capable of indicating when a settlement is likely to occur, and why a settlement occurs in such a high percentage of criminal cases. The model should also be capable of indicating at what amount settlement is likely to occur and what other alternatives are likely to be selected if a settlement does not occur. In showing how the model presented in *Part I* of the article is capable of answering these types of questions, *Part II* will present the general situation and also other situations involving special

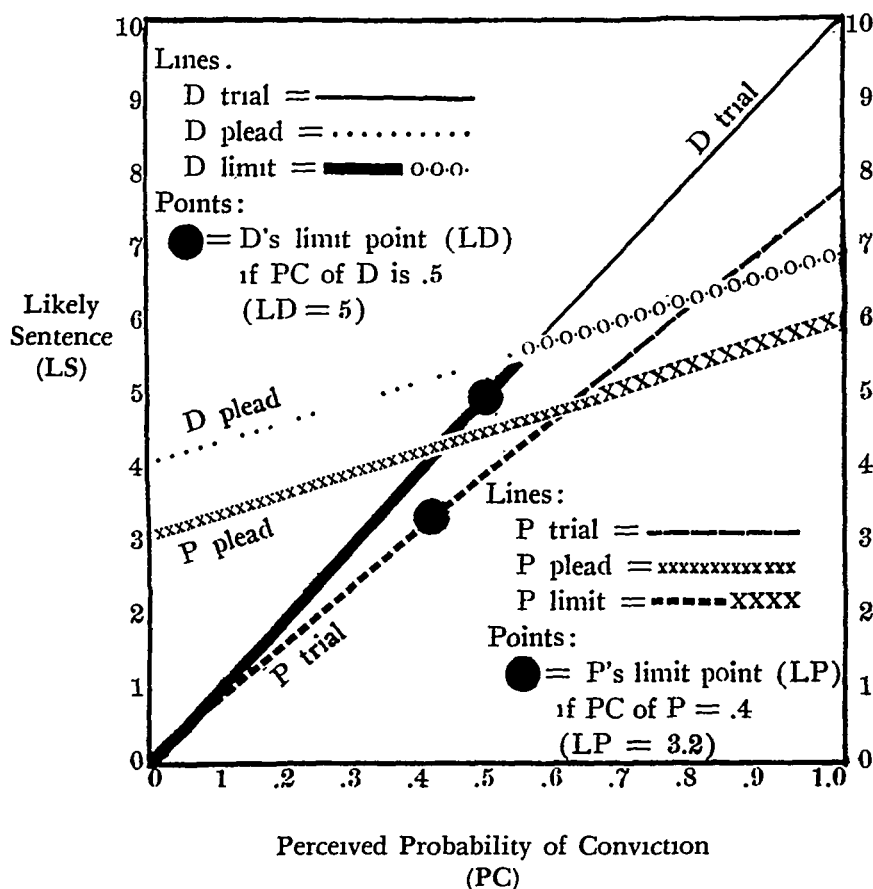
conditions concerning the defendant's strategies toward the alternatives and both parties' degree of knowledge of the contingent probabilities.²

1. General Equilibrium

(a) When convergence is likely to occur

Geometrically speaking, convergence to an equilibrium solution is likely to occur if in a figure like FIGURE 1 the defendant's limit at his perceived probability of conviction (PC) is greater than the prosecutor's

FIGURE 1. THE LIKELY SENTENCES WHICH CORRESPOND TO VARIOUS CONVICTION PROBABILITIES: THE STRATEGIES GRAPH



²Section III-A1, *infra*, on general equilibrium, is based upon Part I, *supra* note 1, Section II-B2(a) (regarding general bargaining limits) and Part I, *supra* note 1, Section II-B3(a) (regarding general PC calculations). Similarly, Section III-A2, *infra*, on special equilibrium, is based upon Part I, *supra* note 1, Section II-B2(b) (regarding special bargaining limits) and Part I, *supra* note 1, Sections II-B3(b) thru II-B3(d) (regarding special PC calculations).

limit at his perceived PC. Thus, in FIGURE 1, if the defendant's PC is .5, his maximum limit is 5 years. If the prosecutor's PC is .4, his maximum limit is 3.2 years. With those facts one can see in FIGURE 1 that the circle corresponding to the defendant's limit is higher than the circle corresponding to the prosecutor's limit. Convergence is likely to occur in that situation because the defendant-buyer is willing to accept a greater sentence than the prosecutor-seller has as his minimum, or the prosecutor-seller is willing to accept a lesser sentence than the defendant-buyer has as his maximum. Using a market analogy, convergence is likely to occur in that situation, because the prosecutor-seller is willing to sell for less than the price at which the defendant-buyer is willing to buy.

On the other hand, if the defendant perceives his PC to be .2, his maximum limit would be 2 years. A circle corresponding to the defendant's limit would then be below the prosecutor's limit of 3.2. In that situation, convergence would be unlikely because the prosecutor would be willing to accept a solution no lower than 3.2 years, and the defendant would accept no sentence higher than 2 years. This assumes of course that sentence maximization and minimization are the goals of the respective parties.

Convergence may not occur not only because the defendant perceives his conviction probability as being substantially lower than the prosecutor's perception of PC but also because the defendant perceives his payoff cells in TABLE 1 to be substantially less than those payoffs perceived by the prosecutor. Thus, even if both the defendant and the prosecutor perceive PC to be .4, there will be no convergence if the defendant perceives that his maximum sentence on being convicted at trial (cell *d*) would be 5 years. At a PC of .4, the defendant perceives that his likely sentence would be only 2

TABLE 1. THE PAYOFF MATRICES AS PERCEIVED BY A DEFENDANT AND A PROSECUTOR

1A. A DEFENDANT'S PAYOFF MATRIX

		Probability of D Being Convicted (PC)	
		0	1.0
Alternative Decisions of D	D Pleads Guilty before a Judge without Bargain (Alt. #2)	a 4	b 7
	D Goes to Trial (Alt. #1)	c 0	d 10

Cells indicate likely sentences (LS) in years as perceived by a hypothetical defendant (D).

1B. A PROSECUTOR'S PAYOFF MATRIX

		Probability of D Being Convicted (PC)	
		0	1.0
Alternative Decisions of D	D Pleads Guilty before a Judge without Bargain (Alt. #2)	a 3	b 6
	D Goes to Trial (Alt. #1)	c 0	d 8

Cells indicate likely sentences (LS) in years as perceived by a hypothetical prosecutor (P).

years since $LS_1 = 0 + 5(.4) = 2$ years.³ Thus, the defendant's 2 year maximum would be below the prosecutor's 3.2 year minimum.

Plea bargaining is in a sense a non-zero sum game since both parties are likely to come out ahead of their fall-back limits. In other words, when a plea bargain is struck, the defendant is getting more satisfaction out of the waiver which the prosecutor gives him of both trial and unbargained judicial pleading than the years he is giving up, since without that waiver he anticipates he would give up even more years. Similarly, the prosecutor is getting more satisfaction from the years the defendant gives him than the waiver or sentence recommendation since he anticipates he would get even less years if the case were resolved at trial or before a judge by a non-negotiated plea bargain. Plea bargaining may be less fruitfully viewed as a zero sum game, in which whatever the defendant gives up the prosecutor gains. The years paid by the defendant are years received by the prosecutor in the same way a price is paid and received for merchandise in our buyer-seller analogy. Perhaps though plea bargaining should be analyzed as being neither a non-zero sum game nor a zero sum game, but rather a game against nature in which both parties are trying to outguess the contingent probabilities and cell payoffs rather than outguess each other. Nevertheless, they probably do try to confuse each other by bluffing. From a methodological perspective, one nice thing about the plea bargaining situation is that it enables one to draw simultaneously upon concepts and methods from the theory of games, decisions, bargains, static equilibrium, and dynamic equilibrium.⁴

³See Part I, *supra* note 1, Section I-B1.

⁴For an example of a model that views plea bargaining in game theory terms see the forthcoming dissertation of Ivan Orton in the political science department at the University of Texas [excerpts on file at the INDIANA LAW JOURNAL]. He views the prosecutor as a threat maker analogous to a blackmailer who can accept the defendant's payment or punish him. He also views the defendant as the victim of a blackmailer who can either comply with the

Therefore, if LD^5 is greater than LP ,⁶ a settlement is likely to be reached, whereas if LD is less than LP , settlement is unlikely to be reached, unless the adjustments for non-sentence goals cause ALD to be greater than ALP .⁷ Similarly, if LD minus LP in a second situation is positive and greater than LD minus LP in a first example, then the likelihood of a settlement is greater in the second situation. The model, however, does not provide a way of assigning probabilities to the likelihood of settlement, because the degree of probability of a settlement when LD is greater than LP depends on the bluffing activities of the parties which, unlike LD and LP , are not predictable from the basic payoff and PC perceptions of the parties. More will be said about the dynamics of bluffing after further discussing the likely equilibrium (in general and under special conditions) without considering bluffing elements.

(b) Results of convergence and non-convergence

When convergence does occur (meaning LD is likely to be higher than LP), the settlement point will, generally, be near the midpoint between LD and LP in the absence of any additional information concerning the bargaining methods of the parties. In a specific case, one side may have the ability to bargain or bluff the other side closer to the other side's limit.⁸ In a large number of cases, however, with approximately equal bargainers, the midpoint should be reasonably accurate. Where S^* is the likely sentence at the point of equilibrium, then S^* should generally equal $.5(LD + LP)$, provided that LD is greater than LP .⁹

What happens, though, if LD is not greater than LP in the solution $S^* = .5(LD + LP)$? The answer can be best understood by looking at FIGURE 1. In the example where the defendant had a PC of .2 and thus a 2 year limit, and the prosecutor had a PC of .4 and thus a 3.2 year limit, the negotiations would break off unless the parties changed their PC perceptions or their payoff perceptions. Upon breaking off the negotiations, the defendant

blackmailer's demands or resist them. These dichotomous positions for each side yield a four-cell payoff matrix. Orton also views the defendant as analogous to a blackmailer and the prosecutor as the victim of a blackmailer, yielding another four-cell payoff matrix. These two matrices are manipulated to provide some insights into the plea bargaining relations between prosecutors and defendants, although on a verbal rather than a quantitative level. His basic idea comes from Ellsberg, *The Theory and Practice of Blackmail* (unpublished paper written at Harvard University, 1961).

⁵ LD is defined to be the bargaining limit of the defendant. See Part I, *supra* note 1, Section II-B1.

⁶ LP is defined to be the bargaining limit of the prosecutor. See Part I, *supra* note 1, Section II-B1.

⁷ ALD and ALP are the adjusted bargaining limits of the defendant and prosecutor, respectively. See Part I, *supra* note 1, Section II-B4(b).

⁸At the midpoint between LD and LP , the gain of the defendant under LD is equal to the gain of the prosecutor over LP . In other words, LD minus S^* (where S^* is the settlement sentence) equals S^* minus LP . If both parties have equal bluffing power, their gains from S^* should be equal. See A. RAPPAPORT, *TWO-PERSON GAME THEORY: THE ESSENTIAL IDEAS* 94-122 (1966) (especially 109 and 120).

⁹See note 7, *supra*.

would proceed to go to trial since going to trial at his PC of .2 is his best alternative decision.¹⁰

If, however, the defendant had a PC of .6, that would mean he would in effect perceive his likely sentence as being 5.8 years since $LS_2 = 4 + (7 - 4)(.6) = 5.8$ years.¹¹ His payoff matrix from TABLE 1, as graphed in FIGURE 1, indicates that he perceives he could, on average, obtain a sentence of 5.8 years by pleading guilty before a judge in a non-negotiated plea. At a PC of .6 the defendant would not want to go to trial because trial would produce, on average, a 6 years sentence ($LS_1 = 0 + 10(.6) = 6$ years).¹² In fact, given the defendant's payoff perceptions, he would prefer to plead guilty before a judge rather than go to trial whenever his PC is greater than .57.¹³ Suppose further that the prosecutor perceives PC to be 1.0, and thus his minimum limit (LP) would be 6 years, the likely or average sentence he perceives the defendant would get from a guilty plea before a judge at a PC of 1.0. Thus, in this hypothetical situation, there would be no convergence because LP is greater than LD. Unlike the previous hypothetical situation, however, the defendant would plead guilty before the judge rather than go to trial when negotiations break off with the prosecutor. This alternative, which presumes that the defendant perceives he can receive a different sentence by pleading guilty before a judge than by plea bargaining with the prosecutor, may not be the case in all jurisdictions or in all cases in the same jurisdiction.

The overall algebraic or symbolic solution to the location of the S* equilibrium point is thus summarized in the following three convergence rules:

1. If LD is greater than or equal to LP, then $S^* = .5(LD + LP)$.
2. If LD is less than LP (meaning convergence unlikely), and the defendant's LS_1 (likely sentence upon going to trial) is greater than his LS_2 (likely sentence from a non-negotiated plea), then the defendant will plead guilty before a judge in a non-negotiated plea.
3. If LD is less than LP, and the defendant's LS_1 is less than his LS_2 , then the defendant will go to trial.

Note that S* represents the likely sentence or settlement which arises from plea bargaining when there is a convergence. The likely sentence from trial or from a guilty plea before a judge is unknown with the given data.¹⁴ This is so, because the basic data as given in TABLE 1 merely shows

¹⁰Part I, *supra* note 1, Section II-A1.

¹¹ LS_2 is defined to be the likely sentence from pleading guilty before the judge in a non-negotiated plea. See Part I, *supra* note 1, Section II-B1.

¹² LS_1 is the likely sentence from going to trial. *Id.*

¹³See Part I, *supra* note 1, Section II-B2(c).

¹⁴The true sentence if the defendant goes to trial (known before trial only to an omniscient being) can be symbolized S' (S primed). If the legal system is a just legal system, then S' should also bear a close relation to the sentence that would be given by an omnibenevolent being. The extent to which plea bargaining tends to arrive at such a sentence is discussed in Section IV-B2, *infra*, which deals with the policy implications of the plea bargaining model.

what the defendant and the prosecutor perceive the payoffs to be, not what the payoffs in fact are, as known only to an omniscient being. Even if the perceived PC's of the parties were averaged in order to derive a better prediction of the probability of conviction, the true probability of conviction would still not be known. In other words, this plea bargaining model is not a judicial decision-making model, although one might try to predict payoff cells¹⁵ and conviction probabilities.¹⁶

(c) Why convergence occurs so frequently

How does the model explain why such a high percentage of criminal cases are settled through plea bargaining? The explanation is probably not caused by defendants perceiving the payoff cells or conviction probabilities as being higher than do prosecutors. There are good reasons for thinking defendants might perceive the situation as being more severe than the prosecutor does (such as awareness of his own guilt and of aggravating circumstances).¹⁷ Similarly, the defendant might also perceive the situation as being less severe (such as wishful thinking based on having more at stake than the prosecutor does). These reasons tend to neutralize each other. Indeed, an empirical survey might reveal that the limit lines of defense counsel and prosecutors as well as their PC's tend to be approximately the same in a given case or set of hypothetical facts, assuming only sentence minimization and maximization are involved.

What propels the defendant and the prosecutor toward equilibrium convergence is the fact that sentence minimization and maximization are not the only goals present in plea bargaining. The defendant may have other goals which will tend to raise his unadjusted LD.¹⁸ For example, a defendant will increase his limit for his litigation costs, including (1) the cost of imprisonment pending trial if the defendant cannot afford bail, (2) the cost of hiring an attorney if the defendant is not poor enough to receive a court appointed attorney, but is still unable to easily absorb expensive attorney fees, and (3) the cost to one's reputation where one is sensitive to adverse publicity. Likewise, the prosecutor's other goals tend to reduce his unadjusted LD. His litigation costs include (1) his limited budget, which prohibits taking all cases to trial, (2) the pressures to reduce court congestion, and (3) the pressures to build a record with a high percentage of convictions.

In other words, the defendant is willing to add a bonus on his LD maximum limit, and the prosecutor is willing to deduct a discount from his LP minimum limit. Thus, even if LD equals LP in a given case, those adjustments are likely to make ALD substantially higher than ALP. The three convergence rules previously given should therefore be adjusted so

¹⁵See Part I, *supra* note 1, Section II-A2.

¹⁶See Part I, *supra* note 1, Section II-B3.

¹⁷See Part I, *supra* note 1, Section II-A1.

¹⁸See Part I, *supra* note 1, Section II-B4.

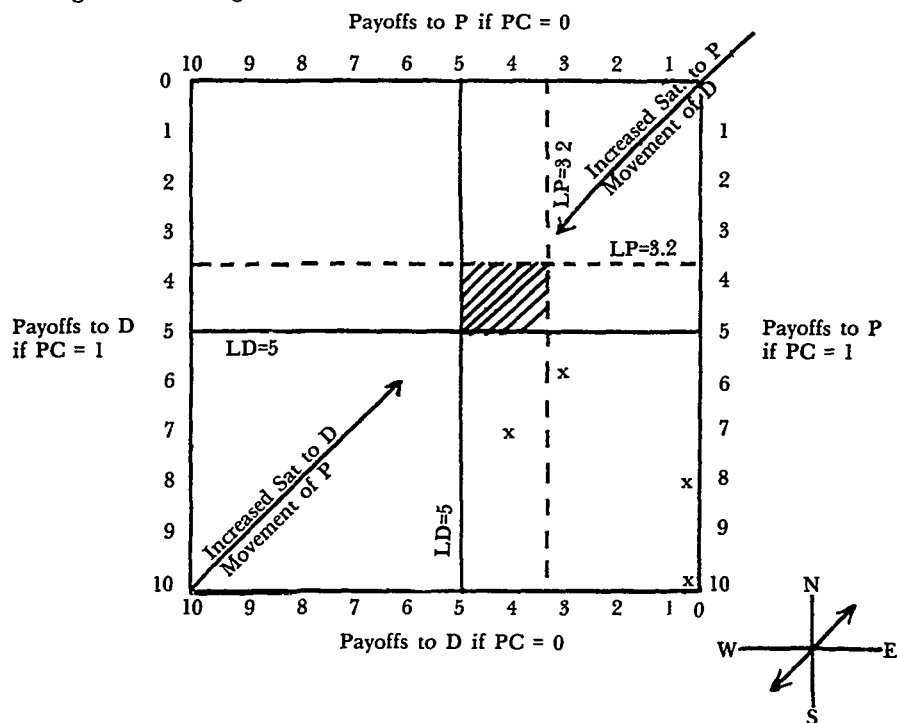
that ALD (or limit of the defendant adjusted for non-sentence goals) is substituted for LD.¹⁹ Similarly, wherever those rules say LP, ALP (or limit of the prosecutor adjusted for non-sentence goals) should be used. Most of the cases are likely to follow convergence rule 1 rather than non-convergence rules 2 and 3, since ALD is likely to be greater than ALP a high percentage of the time.²⁰ As a result, most criminal cases are settled through the plea bargaining process.

The exceptional case is the case where the bargaining settlement costs are greater than the litigation costs. This may be true from the point of view of the defendant in traffic violations and many minor misdemeanor cases like city ordinance violations. In those cases, the defendant may consider it more expensive to plea bargain with a prosecutor than to simply plead guilty before a judge. The settlement costs may outweigh the litigation costs from the point of view of the prosecutor at the other end of the seriousness continuum where, for example, a heinous child murder is involved. In that kind of a case, the prosecutor may feel he has more to lose politically by settling for a reduced charge or sentence than by expending the time and money in trial.²¹

¹⁹See notes 13-14, *supra*, & text accompanying.

²⁰See Part I, *supra* note 1, Section II-B4.

²¹An alternative perspective to FIGURE 1 for analyzing the general equilibrium situation is the Edgeworth box diagram which is like that shown below:



2. Equilibrium Under Special Conditions

In order to further clarify the kind of equilibrium, convergence, or settlement point, if any, that is likely to be produced by plea bargaining, the nature of the equilibrium should be discussed, since the parties may have different strategies toward the alternatives and different degrees of knowledge of the conviction probabilities.

(a) The limits matrix

TABLE 2 shows the bargaining limits for various kinds of defendants and prosecutors, depending on how they are positioned on two dimensions. The first dimension relates to strategies toward the alternative decisions. It includes (1) defendants who only see their trial line, possibly because they are maximax strategists, (2) defendants who only see their plead line, possibly because they are minimax strategists, and (3) defendants who are mixed strategists and thus see both lines in their strategies graph and both rows in their payoff matrix.²²

The continuous lines show that the defendant's bargaining limit is 5 years, and the dashed lines show that the prosecutor's bargaining limit is 3.2 years. The defendant obtains increased satisfaction by moving across indifference curves or lines toward the northeast, whereas the prosecutor obtains increased satisfaction by moving across indifference or equal-satisfaction lines toward the southwest. In their bargaining, the defendant moves from near the northeast corner toward the southwest, and the prosecutor moves from near the southwest corner toward the northeast. If they both get into the shaded area which is the feasible region, an agreement will be reached.

This perspective, however, provides less information than FIGURE 1 (*see also Part I, supra* note 1, Section II-B) because this perspective only provides one limit point for the defendant and one for the prosecutor, since FIGURE 1 does not show probabilities of conviction on any axis. It also provides less information than FIGURE 2 (Section III-B, *infra*), since it does not show time stages on any axis. The main value of the Edgeworth box perspective is that it can show degrees of risk preference or risk avoidance as non-sentence goals by the shape of the indifference curves which pass through the defendant's payoff points at 0,10 for going to trial and 4, 7 for pleading before a judge, or which pass through the prosecutor's payoff points at 0,8 for going to trial and 3, 6 for pleading before a judge. If the defendant's trial point is on a higher, lower, or the same indifference curve as his pleading point, then he is a risk preferrer, risk avoider, or risk neutral respectively. For further detail on this perspective, *see* W. BAUMOL, *ECONOMIC THEORY AND OPERATIONS ANALYSIS* (1965); J. CROSS, *ECONOMICS OF BARGAINING* (1969). The Edgeworth box perspective would be more useful if the two goods being exchanged were both intervally measured and were shown on each axis. The defendant, however, is paying years in jail to the prosecutor (which can be intervally measured) in return for a waiver of litigation (which is a yes-no dichotomy).

²²A defendant may also only see his trial line not because he is a maximax strategist but because it is the only line to see in some places or cases where pleading guilty before the judge is not a meaningful alternative to going to trial or to plea bargaining with the prosecutor. A defendant may also only see his plead line not because he is a minimax strategist but because for reasons of time, money or stigma he just cannot possibly consider going to trial. The first dimension also includes prosecutors who perceive a given defendant to be a maximax strategist, a minimax strategist, or a mixed strategist. *See Part I, supra* note 1, Section II-B2.

The second dimension relates to conditions of knowledge toward PC. It includes defendants or prosecutors (1) who are certain of either acquittal or conviction, (2) who are totally ignorant of what PC might be, and (3) who think PC is at some fairly precise risk point between $PC = 0$ and $PC = 1.0$. This third category includes those parties who think of PC in terms of a range but who tend to round off to the lower PC boundary, the midpoint, or the upper PC boundary, depending on whether they are optimistic, middling, or pessimistic.

The numbers in the cells of TABLE 2 indicate the upper limit for the defendant and the lower limit for the prosecutor depending on how each party is positioned on those two dimensions. For example, in the cell in the upper lefthand corner, the limit of the defendant who is certain that he will be acquitted, viewing trial as the only meaningful alternative to plea bargaining is shown. Such a defendant will not accept an offer from the prosecutor unless the offer is at or below the defendant's limit of zero years in jail, assuming he only wants to minimize his sentence. In other words, the cells do not show what bonus should be added to indicate the non-sentence benefits received by the defendant.

As an example, at the opposite end of TABLE 2, in the cell in the lower right-hand corner, the limit of the prosecutor who perceives the probability of conviction at .4 and who perceives the defendant as working with the two alternatives to plea bargaining, namely going to trial or pleading guilty before a judge, is shown. Such a prosecutor will not accept an offer from the defendant unless the offer is at or above 3.2 years. The 3.2 years is the expected sentence from a trial at a PC of .4 given the cell payoffs as perceived by the prosecutor. The prosecutor perceives the defendant as being more likely to plead guilty before a judge when PC is .4 because pleading guilty before a judge at a PC of .4 is perceived as producing an expected sentence of 4.2 years.

At the left side of the graph are shown the LD's and LP's for defendants and prosecutors certain of acquittal ($PC = 0$) or conviction ($PC = 1.0$). In the middle of the TABLE, the LD's and LP's when the reasonable range of PC is the total range between 0 and 1.0 are also shown. In the latter situation, the LD depends on whether the defendant is optimistic ($PC = 0$), middling ($PC = .5$), or pessimistic ($PC = 1.0$). On the other hand, the LP depends on whether the prosecutor is optimistic ($PC = 1.0$), middling ($PC = .5$), or pessimistic ($PC = 0$). On the right side of the TABLE, the LD's of a defendant who perceives PC at .5 and the LP's of a prosecutor who perceives PC at .4 are shown. A separate LD and LP is shown in all three parts of the TABLE depending on whether the defendant sees only the trial line, the plead line, or both lines.²³

²³In addition to stating the number of years that corresponds to each type of defendant and prosecutor, the table also gives the formula that was used to calculate the LD or LP years. The formulas are stated in terms of the defendant's or the prosecutor's *a*, *b*, *c*, *d* perceived cell payoffs. All the formulas in the first and second thirds of the table on the left side are

In TABLE 2, there are many types of defendants and many types of prosecutors. There are in fact three types of defendants who operate under a condition of risk, namely a maximax risk defendant, a minimax risk defendant, and a mixed strategy risk defendant. In addition, there are nine types of defendants who operate under conditions of ignorance since there are three strategies corresponding to each of the three optimism-pessimism points. Moreover, there are six types of defendants operating under conditions of certainty since there are two conditions of certainty and three strategies toward the alternatives.

Even though there are eighteen possible types of defendants and prosecutors shown in TABLE 2, this does not necessarily mean that all types correspond to defendants and prosecutors who frequently exist, and especially not in equal numbers. For example, there are at least two defendant types that probably represent null classes. One is the defendant who sees only the plead line even though he is certain of acquittal. Any defendant who is certain of acquittal is unlikely to consider pleading guilty, unless a minor traffic or parking violation is involved. This "unless" limitation is not true for the hypothetical felony for which the defendant could conceivably receive at least ten years maximum penalty. The other null class is the defendant who sees only the trial line even though he is certain of conviction. Any defendant who is certain of conviction is unlikely to go to trial where his sentence is likely to be higher than pleading guilty before a judge.²⁴ The most common situations in TABLE 2 are probably conditions of risk (*i.e.* the right side) with defendants pursuing a mixed strategy that involves considering both the trial line and the plead line (*i.e.* the bottom row possibilities).

(b) The results matrix

TABLE 3 shows the results of clashes between certain types of defendants and prosecutors. If each of the eighteen types of defendants were pitted against the eighteen types of prosecutors shown in TABLE 2, 324 scenarios would be generated which is a rather large number of clashes to show in one results table. To make the TABLE more manageable, TABLE 3 just deals with eight types of defendants and eight types of prosecutors (and thus 64 scenarios) by dealing only with the middling optimism-pessimism type of party under conditions of ignorance, and only with the mixed strategist type of party under conditions of certainty. The reader can stage any of the remaining scenarios if he wishes to do so.

simplified versions of the formulas in the last third of the table on the right side. For example, the optimistic prosecutor who lacks any knowledge of PC and who operates in a jurisdiction where going to trial is the only alternative to plea bargaining has an LP of 8 years based on cell *d*. That 8 could also be calculated from the formula $LS_1 = 0 + (8-0) (1.0)$, or from the formula $LS_1 = (1-1.0)0 + (1.0)8$, both of which are given in the last third of the table. To review what is involved in calculating an LD or an LP, the reader can check the calculations for any or all of the cells in TABLE 2 using the raw data from TABLE 1 and the graphical approach of FIGURE 1.

²⁴The defendant will not plead guilty if he knows the judge will give him a more severe sentence than will the jury in those unusual places where juries can determine sentence. See Part I, *supra* note 1, Seciton II-Á1.

TABLE 3. RESULTS OF CLASHES BETWEEN CERTAIN TYPES OF DEFENDANTS
AND PROSECUTORS: THE RESULTS MATRIX

Type of Defendant		Type of Prosecutor							
		Certainty (Just Both Lines)		Ignorance (Only Midding O-P)			Risk (Only PC=4)		
		PC=0	PC=1	Trial Line	Plead Line	Both Lines	Trial Line	Plead Line	Both Lines
Certainty (Just Both Lines)	PC=0	0	0	0	T	4	0	T	0
	PC=1	7	3.5	7	5.5	4	7	5.1	7
Ignorance (Only Midding O-P)	Trial Line	5	2.5	5	4.5	4	5	4.1	5
	Plead Line	5.5	2.75	5.5	4.75	4	5.5	4.35	5.5
	Both Lines	5	2.5	5	4.5	4	5	4.1	5
	Risk	5	2.5	5	4.5	4	5	4.1	5
Risk (Only PC=5)	Trial Line	5	2.5	5	4.5	4	5	4.1	5
	Plead Line	5	2.75	5.5	4.75	4	5.5	4.35	5.5
	Both Lines	5	2.5	5	4.5	4	5	4.1	5
	Risk	5	2.5	5	4.5	4	5	4.1	5

Key:

LD
R
LP

R = Result which can be S*, T, J, or F

Conditions of convergence:

1. Convergence if $LD \geq LP$
2. Non convergence if $LD < LP$

Results of Convergence: $S^* = .5 (LD + LP)$

Results of Non-Convergence:

1. T = Sentence determined by D going to trial. $LD < 5.71$.
2. J = Sentence determined by D going to judge and pleading guilty. $LD > 5.71$.
3. F = Coin flip or related random method will determine whether D goes to trial or pleads before judge. $LD = 5.71$.

To read TABLE 3, find the cell corresponding to any particular clash between a hypothetical defendant and prosecutor. Take for instance the cell in the lower right-hand corner where each party is operating under conditions of risk and each one is considering both the trial alternative and the plead alternative. In that scenario, the hypothetical defendant has an upper limit (LD) of 5 years, and our hypothetical prosecutor has a lower limit (LP) of 3.2 years. Since the defendant's upper limit is greater than the prosecutor's lower limit, there is likely to be convergence, and the likely convergence point (S^*) should be near the midpoint between LD and LP (*i.e.* 4.1).

Some of the scenarios, on the other hand, involve an LD that is lower than the LP for that scenario. For example, if the defendant, given his perceived cell payoffs and his perceived PC of .5, considers both the trial and plead lines or alternatives, then he will have an LD of 5. If, however, the prosecutor is certain the defendant will be convicted and the prosecutor also considers both lines, then the prosecutor will have an LP of 6 as shown in column two of the bottom row. Therefore, convergence will not occur, and the defendant will resort to either trial or pleading guilty before a judge. Which alternative he chooses will depend on whether LD is greater or less than LS^* , which is the likely sentence at the point where LS_1 equals LS_2 . In this specific hypothetical situation, since LD is 5 and LS^* is 5.71, the defendant will go to trial as his alternative to a settlement through plea bargaining.

There are no cells in our TABLE 3 where non-convergence was resolved by the defendant electing to plead guilty before a judge. A hypothetical situation could be created, though, where such a resolution would have occurred in our results matrix. A defendant with the same payoffs as TABLE 1 but who perceives his probability of conviction as being greater than .57 rather than just .5 would elect to plead guilty before a judge. In that situation, if LD is less than LP, the hypothetical defendant will elect a guilty plea in order to minimize his sentence.²⁵ To the extent that

²⁵We could have also had some J's in TABLE 3 by changing the defendant's payoff cells so that with a PC of .5 or even lower, his LD would be greater than his LS^* . Doing so would involve decreasing cells *a* and *b* and/or increasing cells *c* and *d*. TABLE 3 would also have some J's in it if we had included the pessimistic defendant operating under conditions of ignorance rather than just the middle defendant, since the pessimistic defendant perceives PC to be 1.0.

There seems to be no empirical data available indicating what percentage of the time the defendant turns to the judge with a non-bargained guilty plea rather than go to trial when plea bargaining negotiations break down. Donald Newman's data indicates that in the Wisconsin county he studied, six percent of all the felony cases went to trial, 56 percent were settled by plea bargaining (*i.e.* .60 times .94), and 38 percent were settled by non-bargained guilty pleas (*i.e.* .40 times .94). See Part I, *supra* note 1, Section I. However, the data does not indicate (1) what proportion of the 38 percent involving guilty pleas never involved plea bargaining, (2) what proportion did involve plea bargaining that broke down, (3) what proportion of the 6 percent that went to trial never involved plea bargaining, and (4) what proportion did involve plea bargaining that broke down. To determine the most common alternative occurrence when plea bargaining breaks down, one must compare proportions (2) times .38 and (4) times .06. Writers on plea bargaining often assume that if plea bargaining

defendants tend to think their conviction probabilities are low partly explains why defendants may be more likely to resort to trial as an alternative to plea bargaining. Part of the explanation may also relate to the fact that in many jurisdictions judges ask for the prosecutor's sentencing recommendation if the defendant pleads guilty, and thus the judge may not serve as a sufficiently independent alternative to the prosecutor.²⁶

Both the defendant and the prosecutor would like to know what the other side's payoff cells, PC perceptions, and thus bargaining limits are so that each side could strike a bargain that will maximize his side's gain and minimize the other side's gain, but still obtain convergence. They both, however, try to make their own payoff and PC perceptions, and thus their limits, reflect reality as accurately as possible rather than reflect the other side's possible misperceptions, especially where they have encouraged pessimistic misperceptions by the other side. That kind of bluffing encouragement comes out more clearly in discussing the dynamic equilibrium model.

B. The Dynamics of Converging Toward Equilibrium

1. The Time Path Graph

TABLES 2 and 3 involved the calculation of equilibrium points which are likely to be determined by different types of defendants with different payoff and PC perceptions.²⁷ The discussion highlighted the given data and the results. However, the process whereby one moves from the givens to the results was not discussed. Thus, the equilibrium discussion has so far

were made more difficult, the quantity of trials would increase greatly causing the criminal justice system to collapse. Such writers tend to think that the only alternative to plea bargaining is to go to trial rather than to plead guilty before a judge who is likely to give a lower sentence after a guilty plea than after a trial conviction. See PRESIDENT'S COMMISSION ON LAW ENFORCEMENT AND ADMINISTRATION OF JUSTICE, TASK FORCE REPORT: THE COURTS 10 (1967); Hoffman, *Plea Bargaining and the Role of the Judge*, 53 F.R.D. 499 (1971); Landes, *An Economic Approach of the Courts*, 14 J.L. & ECON. 61 (1971). Pleading guilty before a judge without plea bargaining consumes some judicial resources, but not nearly so much as a trial.

²⁶In order to simplify the arithmetic in the example by using a defendant with a PC of .5, some repetition was introduced in TABLE 3 and TABLE 2. If the defendant was ignorant of PC and middling on the optimism-pessimism scale he had the same limit as if he was knowledgeable that PC equals .5 in his case. It is not artificial repetition, however, that on any given row in TABLE 3 the LD of the defendant remains the same. That is so because neither the payoff cells nor the LD of the defendant is changed by the type of prosecutor with which he is dealing, except in the sense that the more competent the prosecutor, the higher the defendant should perceive PC to be, and the more severe and influential the prosecutor, the higher the defendant should perceive the payoffs to be. Similarly, on any given column the LP of the prosecutor remains the same because the LP is not changed by the type of defendant or defense counsel with which he is dealing, except in the sense that the more sympathy-arousing or competent the other side, the lower the prosecutor should perceive PC and the payoff cells to be.

²⁷See Section III-A, *supra*.

been a static rather than a dynamic or process equilibrium. More specifically, it has been a comparative static equilibrium because the equilibrium points produced by different types of situations have been compared. It now seems appropriate to seek to extend the model to explain in simple arithmetic terms the process of moving from the givens to the results.

FIGURE 2 introduces a time dimension on the horizontal axis as contrasted to the probability dimension on the horizontal axis of FIGURE 1. The vertical axes on both figures represent sentence severity or charge severity where there is bargaining over the charge instead of, or in addition to, bargaining over the sentence. In FIGURE 1, however, the vertical axis represents years likely to be received at different conviction probabilities, whereas in FIGURE 2 the vertical axis represents years offered by the defendant or the prosecutor at different stages in the negotiation process. FIGURE 2 is referred to as a time path graph because it shows how converging or diverging variables change over time. The variables in this situation are the defendant's offers and the prosecutor's offers.

Most of the data used in FIGURE 2 comes from the payoff matrices of TABLE 1, the bargaining limits of FIGURE 1 and TABLE 2, and from the previous discussion of the defendant's bonus factor and the prosecutor's discount factor.²⁸

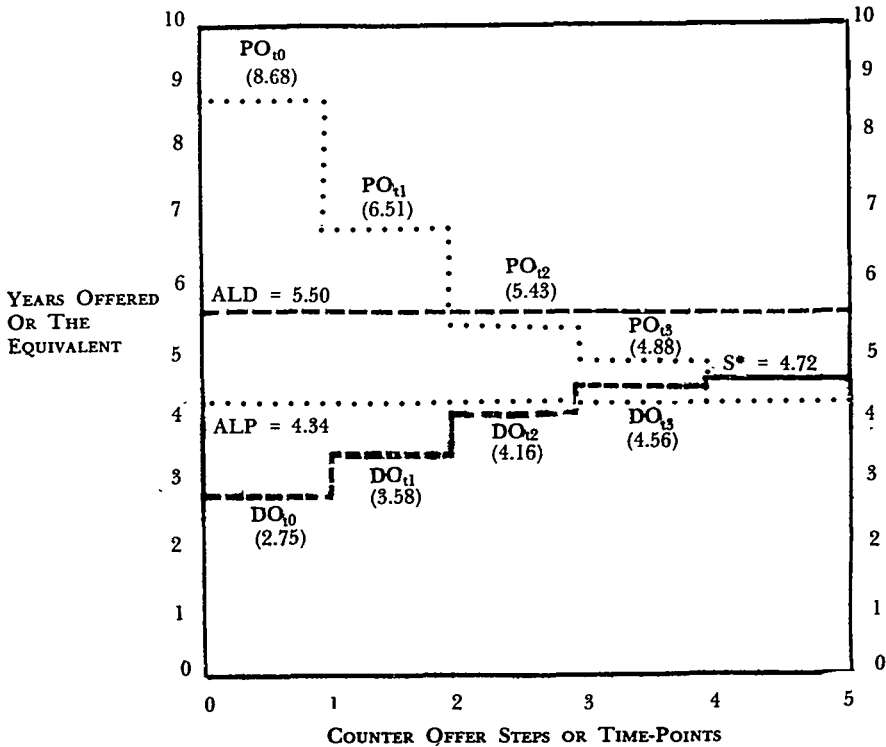
The only change from the previous examples which we used is to set the prosecutor's perceived probability of conviction at .7 rather than .4. This has the effect of making the prosecutor and the defendant initially further apart, so that the convergence process will occur more slowly for observation. As indicated at the bottom of FIGURE 2, the defendant's upper limit is 5 years when only considering sentence minimization and 5½ years when adjusted by a ten percent bonus to consider other goals. Similarly, the prosecutor's lower limit is 5.1 years when only considering sentence maximization and 4.34 years when adjusted by a fifteen percent discount to consider other goals. Over the time points in the dynamic bargaining process, either ALD or ALP can change as a result of new information or new values. For the sake of simplicity, however, FIGURE 2 shows ALD and ALP as being constant across the graph.

2. Initial Offers and Bluffing

In order to understand FIGURE 2 better, it is necessary to introduce two new concepts which intervene between the givens of TABLE 2 and the results of TABLE 3. The concepts are initial offer and counter offer. These concepts are easier to understand if we recall the analogy of the defendant (or prosecutor) to a buyer (or seller) who is seeking as low (or high) a price as possible in a bargaining bazaar without fixed prices. Under those circumstances, the initial offer of the defendant would logically be lower

²⁸See Part I, *supra* note 1, Section II-B4(b).

FIGURE 2. DYNAMIC PLEA BARGAINING FROM INITIAL OFFERS TO COUNTER OFFERS TO EQUILIBRIUM: THE TIME PATH GRAPH



DEFENDANT'S BARGAINING PICTURE

Given:

$a = 4$, $b = 7$, $c = 0$, $d = 10$, $PC = .5$,
 $\%XD = .10$, $EF = .5$, $RD = .3$

Calculations:

$$PC^* = (a-c)/(a-b-c+d) = (4-0)/(4-7-0+10) = .57$$

$$LD = LS_1 \text{ since } PC < PC^*$$

$$LD = LS_1 = c + (d-c)PC = 0 + (10-0) \cdot .5 = 5$$

$$ALD = LD + (\%XD \cdot LD) = 5 + (.10 \cdot 5) = 5.5$$

$$O_{10} = EF \cdot ALD = .5(5.5) = 2.75$$

$$O_{11} = O_{10} + RD(ALD - O_{10}) = 2.75 + .3(5.5 - 2.75) = 3.58$$

PROSECUTOR'S BARGAINING PICTURE

Given:

$a = 3$, $b = 6$, $c = 0$, $d = 8$, $PC = .7$,
 $\%XP = .15$, $EF = 2$, $RP = .5$

Calculations:

$$PC^* = (a-c)/(a-b-c+d) = (3-0)/(3-6-0+8) = .60$$

$$LP = LS_2 \text{ since } PC > PC^*$$

$$LP = LS_2 = a + (b-c)PC = 3 + (6-3) \cdot .7 = 5.1$$

$$ALP = LP - (\%XP \cdot LP) = 5.1 - (.15 \cdot 5.1) = 4.34$$

$$O_{10} = EF \cdot ALP = 2(4.34) = 8.68$$

$$O_{11} = O_{10} - RP(O_{10} - ALP) = 8.68 - .5(8.68 - 4.34) = 6.51$$

than the limit he is finally willing to accept. Similarly, the initial offer of the prosecutor would logically be higher than the limit he is willing to accept. A better understanding of the nature of initial offers in plea bargaining is quite helpful in understanding the dynamics of going from the inputs to the outputs of the bargaining process, especially with regard to bluffing or exaggerating one's statements about outer limits, the likely sentence, or the probability of conviction.

(a) Calculating initial offers

The defendant's initial offer should be calculated by multiplying his limit by some fraction less than one. That decimal can be called the defendant's exaggeration factor, since it represents a coefficient of the degree to which he exaggerates the lowness of the limit. Multiplication of the defendant's limit by such a decimal in effect reduces the defendant's upper limit to indicate his low initial position. For the want of better information,²⁹ assume the hypothetical defendant has an exaggeration factor of .5, meaning his initial offer is one-half of his bargaining limit.

The size of the defendant's exaggeration factor (EF) depends partly on the psychology of his bluffing strategy. If the defendant sets his exaggeration factor at .1 or an extremely low point, he may cause the prosecutor to consider him unreasonable. The prosecutor may then break off negotiations even though the defendant may really have been willing to settle at a mutually good bargain. On the other hand, if the defendant sets his exaggeration factor at .9 or an extremely high point, he may yield too much just to obtain an agreement. Where within this range the defendant sets his exaggeration factor depends partly on what he perceives the prosecutor's likely reaction to be, although the defendant may have an opportunity to remedy an unduly high or unduly low exaggeration factor by compensating on his first counter-offer. The defendant's credibility, however, will be disrupted if his first counter offer is a lot higher than his initial offer in order to compensate for an unduly low initial offer. Similarly, the defendant's reasonableness or good faith will be put in doubt if his first counter offer involves a backward move or trivial difference upward from his initial offer in order to compensate for an unduly high initial offer. The credibility of an offer refers to telling the truth when stating one's limits. The reasonableness or good faith of an offer refers to being willing to make concessions.³⁰

The point at which the defendant sets his exaggeration factor, and thus his initial offer, also depends on his willingness to go to trial.³¹ The more

²⁹Perhaps a questionnaire survey aimed at defense counsel would reveal more precisely how much less the average defendant or defense counsel tends to offer in stating his initial bargaining position than he is actually willing to accept.

³⁰For further discussion of the psychology of bluffing and other negotiation techniques, see J. ILICH, *THE ART AND SKILL OF SUCCESSFUL NEGOTIATION* (1973); C. KARRASS, *THE NEGOTIATING GAME* (1970); R. WALTON & R. MCKEESIE, *A BEHAVIORAL THEORY OF LABOR NEGOTIATION* (1965).

³¹His willingness to go to trial is governed by his position on the maximax-minimax dimension regarding his perception of the trial-pleading alternatives, the optimism-pessimism

willing he is to go to trial, the more he will exaggerate the lowness of his bargaining limit, since he is not so concerned with making an offer the prosecutor will eventually accept. In these cases, the defendant is more likely to have an exaggeration factor of .1 rather than .9. Thus, if the adjusted limit is 5 years, the defendant who is more willing to go to trial is more likely to have an initial offer of 6 months rather than 4½ years.

To calculate the prosecutor's initial offer, multiply his limit by some integer or fraction greater than 1. Multiplying the prosecutor's limit by such a number increases the prosecutor's lower limit to indicate his high initial position. For the want of better information,³² the hypothetical prosecutor may be assumed to have an exaggeration factor of two, meaning he tends to double his initial offer. Like the defendant, the exact position of the prosecutor's exaggeration factor depends partly on the psychology of his bluffing strategy in dealing with the defendant and on the prosecutor's willingness to go to trial.³³

The higher the defendant's limit, the higher is his initial offer if his exaggeration is held constant since his initial offer is the product of his exaggeration factor times his limit. The same is true of the prosecutor. If the defendant's limit is high, however, the defendant is also likely to exaggerate more how low his limit is by using a smaller decimal for an exaggeration factor. This may be true because defendants may be more willing to go to trial, particularly to benefit from the safeguards for the innocent, when (1) the crime is more severe, (2) the likely sentence is therefore greater, and (3) the defendant-buyer's bargaining limit or maximum price is thus higher. As previously mentioned, the more willing the defendant is to go to trial, the more he may exaggerate in a downward direction his bargaining limit by setting a low initial offer because he cares less about the negotiations breaking down. If the exaggeration factor is partly determined by ALD or the defendant's limit, then the exaggeration factor is at least partly an endogenous variable, *i.e.* determined by one of the variables which can be calculated, rather than a given or exogenous variable. Nevertheless, for both the defendant and the prosecutor, the exaggeration factor is probably mainly determined by the psychology of bluffing strategies.³⁴

dimension regarding his narrowing of the PC or cell payoff ranges, and the risk preferrer-risk avoidance dimension regarding his attitude toward risk as a non-sentence goal. His willingness to go to trial also varies with the severity of the crime in the sense that the more severe the crime, the less likely the defendant will plead guilty. Similarly, the defendant is more willing to go to trial if his litigation costs are lowered and his settlement costs are raised.

³²See note 29, *supra*.

³³In our hypothetical example, the defendant's exaggeration factor is the reciprocal of the prosecutor's exaggeration factor and vice versa. This is coincidental since they are determined separately although a prosecutor may tend to exaggerate his lower limit more if he perceives that the defendant is highly exaggerating his upper limit and vice versa.

³⁴The splitting rate discussed in Section III-B3, *infra*, is also a function of bluffing psychology. On the mathematics of bluffing, see J. CROSS, *ECONOMICS OF BARGAINING*, 166-80 (1969). As Cross emphasizes and as both our static and dynamic models tend to show, where

Bluffing involves communicating information that the communicator thinks is false to the other party with regard to the communicator's limits or with regard to facts relevant to the sentence payoffs or the conviction probability. Bluffing is not likely to have any effect on whether there will be convergence if bluffing merely understates the defendant's ALD or overstates the prosecutor's ALP at the initial offer stage or a counter offer stage. Whether or not convergence will occur is mainly dependent on whether ALD is greater than ALP. The exception to this rule is where the bluffing is strong enough and persisted in long enough to cause the other side to break off negotiations prematurely. Bluffing as to ALD and ALP, however, can influence the point of convergence, since convergence tends to be at the midpoint between the bluffed or claimed ALD and the bluffed or claimed ALP. Bluffing can especially affect the convergence point if the bluffing causes the defendant to think the payoff sentences and conviction probability are higher than they really are, or if the bluffing causes the prosecutor to think the payoffs and PC are lower than they really are.

If both sides would refrain from engaging in any bluffing or exaggeration, but instead would immediately inform the other side what their respective limits are, then approximately the same agreement or non-agreement could be reached much more quickly, assuming the bluffing on both sides is evenly balanced in degree of exaggeration and credibility. It is, however, unrealistic to expect competitive sides with valuable stakes to be that cooperative and trusting. Likewise, agreement could be reached more quickly (and with results that come closer to the true sentence which would be arrived at through a trial) if the parties would share information with each other concerning the probability of conviction and the sentence payoffs, rather than try to bluff each other into thinking PC and the payoff cells are lower or higher than they really are. Discovery can force a more honest sharing of information.³⁵

Applying the defendant's exaggeration factor of .5 to his adjusted limit of 5½ years, his initial offer should be 2.75 years. That initial offer is symbolized DO_{10} for defendant's offer at time zero. Applying the prosecutor's exaggeration factor of two to his adjusted limit of 4.34 years, his initial offer (PO_{10}) is 8.68 years. One might ask how a prosecutor with any credibility or ethical reasonableness could ask for 8.68 years initially when the defendant "knows," according to FIGURE 2, that the worst that is likely to happen to him in our hypothetical case is that he will get 5 years by going to trial or 5.5 years by pleading guilty before a judge. The answer is

the parties start in their bargaining generally has little effect on the point of agreement (S^*) which is largely determined by the defendant's limit (ALD) and the prosecutor's limit (ALP). In the model, as is clarified in Section III-B2(c), *infra*, the initial offers have no bearing on whether convergence will be reached since that is determined by whether ALD is greater than ALP. The initial offers along with the splitting rates, however, do determine the last counter offers before the parties cross over, and the midpoint between those last counter offers is the likely settlement point. See Section III-B4, *infra*.

³⁵See Section IV-B1, *infra*.

that the defendant really is not absolutely certain that his estimate is correct, since he may be too low in his perception that .5 is his conviction probability, and he may be too low in his perception of the cell payoffs for trial and pleading. Indeed, the prosecutor will try to convince the defendant that his perceptions are too low.³⁶ Similarly, the defendant will try to convince the prosecutor that his perceptions are too high, giving the defendant's initial offer more credibility and reasonableness.

(b) Ordering initial offers

Once the considerations involved in calculating the initial offers have been determined, the next question is who shall make the first initial offer. Who makes the first offer is irrelevant to whether and at what point an equilibrium will be attained, but it is relevant to describing the dynamics of plea bargaining negotiation. Like the exaggeration factor, the order of the initial offers is largely determined by the psychology and personalities of the bargainers³⁷ rather than through the deductive axiomatic reasoning used to arrive at the defendant's limit, the prosecutor's limit, and their equilibrium point.

Nevertheless, perhaps one can say that if the prosecutor perceives PC as being low, he is more likely to make an initial offer to avoid trial than if he perceives PC as being high. Similarly, if the defendant perceives PC as being high he is more likely to make an initial offer to avoid trial than if he perceives PC as being low. A high PC in this context can be defined as one above PC*,³⁸ and a low PC as one below PC*. Normally, both the prosecutor and the defendant would perceive PC as being about equally high or equally low. If, however, the prosecutor perceives PC as being low and the defendant perceives PC as being high, both will want to avoid trial. In that case, the initial offer would probably be made by the side whose perceived PC is closest to his maximum pessimistic position. In other words, if the PC of the prosecutor minus 0.0 is smaller than 1.0 minus the PC of the defendant, then the prosecutor will tend to make the initial offer. Otherwise, the defendant will. More empirically valid procedures could be established for determining which party should make the first offer,³⁹ but for the purposes of this model, this formula should be satisfactory.

Applying the above analysis to the data shown at the bottom of FIGURE 2, the defendant perceives PC as being low, since his perceived PC of .5 is

³⁶The prosecutor's initial offer can still be meaningful even if it is greater than either the defendant's or the prosecutor's perception of cell *d* which shows the likely sentence if the defendant is convicted after a trial. The prosecutor's initial offer cannot be meaningful or ethical, though, if it is greater than the statutory maximum allowed for the crime involved.

³⁷See notes 29-36, *supra*, & text accompanying.

³⁸See Part I, *supra* note 1, Section I-B2(c).

³⁹This matter of who initiates plea bargaining may, however, be more determined by institutionalized procedures for different crime categories than by the evidence in specific cases. We thus have in the ordering of initial offers another side aspect of our model which can be empirically tested through a questionnaire and interviewing survey.

below his PC* of .57. Therefore, the defendant has a preference for trial as compared to pleading guilty before a judge without a bargain. In addition, the prosecutor perceives PC as being high, since his perceived PC of .7 is above his PC* of .60. The prosecutor is also not so anxious to avoid trial since he perceives trial as likely to produce a longer sentence than that produced by pleading guilty before a judge. The defendant, however, is likely to be less enthusiastic about going to trial than the prosecutor is since .5 is only .07 less than .57, whereas .7 is .10 more than .60. Thus, the defendant is more likely to make the initial offer. The same conclusion could be reached by observing that the distance between .5 and 1.0, the worst PC the defendant could have, is less than the distance between .7 and 0.0, the worst PC the prosecutor could have. In other words, since the defendant perceives his trial alternative as less attractive than the prosecutor views his own trial alternative, the defendant is more likely to make the first move toward settlement.

(c) Relation to convergence

It should be noted that one cannot tell whether there will be convergence by simply observing the relative rank or closeness of the initial offers. Convergence occurs if the adjusted limit of the defendant is greater than or equal to the adjusted limit of the prosecutor. Convergence fails and negotiations break down if the adjusted limit of the defendant is less than the adjusted limit of the prosecutor. In either of these situations, however, the initial offer of the prosecutor will be greater than the initial offer of the defendant. Likewise, even though the initial offers of the prosecutor and the defendant may be very far apart, they will still reach convergence if ALD is greater than ALP. The corollary of this is that if the initial offers of the prosecutor and the defendant are very close together, they may still not reach convergence if ALD is smaller than ALP.

The reason the initial offer of the prosecutor is almost always likely to be greater than the initial offer of the defendant is because neither the prosecutor nor the defendant is ever likely to cross over the other in making either their initial offers or their counter offers. In other words, if the prosecutor initially offers seven years, the defendant's first offer is not going to be greater than seven years if he is seeking to minimize his sentence. Likewise, if the defendant's initial offer is three years, the prosecutor's first offer is not going to be less than three years if he is seeking to maximize the sentence. This is true at any stage in the negotiations process.

As a result, given the data in FIGURE 2, the defendant is likely to make the first offer, which should be $2 \frac{3}{4}$ years. The prosecutor is then likely to initially offer about $8 \frac{2}{3}$ years. In a dynamic scenario the defendant is now ready to make his first counter offer.

3. Calculating Counter Offers

A counter offer is any offer made by a party after his initial offer. It

seems reasonable to expect each counter offer of the defendant to be higher than his low initial offer. In other words, the defendant's offer should reasonably be expected to ascend in staircase fashion, although not necessarily in equal jumps, from his initial offer toward his bargaining limit. Likewise, it seems reasonable to expect the prosecutor's offers to descend in staircase fashion from his high initial offer toward his bargaining limit. In bargaining, one normally makes bigger concessions or bigger jumps in the beginning and then smaller concessions as one gets closer toward one's limit or a settlement point. There may be some situations, of course, where either the prosecutor or the defendant backtracks because he feels he has gone too far in light of his new perceptions of PC or the payoffs, or because the other side seems especially willing to concede. Nevertheless, the general trend of the defendant's counter offers is upward, and the prosecutor's counter offers is downward, as shown in the time-path graph of FIGURE 2.

(a) First counter offer

In light of the above, the first counter offer of the defendant equals his initial offer plus an increment. Likewise, the first counter offer of the prosecutor equals his initial offer minus a decrement. The increment for the defendant equals a portion of the distance between his last offer and his bargaining limit. The decrement for the prosecutor equals a portion of the distance between his last offer and his bargaining limit.⁴⁰ The portion of that distance for either the defendant or the prosecutor is a decimal less than one assuming the defendant or the prosecutor do not jump to their bargaining limit. That decimal or "splitting rate" can be symbolized RD for the defendant and RP for the prosecutor.⁴¹

The splitting rates for the defendant and the prosecutor are determined by the same kind of considerations with regard to the psychology of bluffing strategy and one's willingness to go to trial that determined the exaggeration factors.⁴² For lack of any better information, assume the defendant has a splitting rate of .3 and the prosecutor has a splitting rate of

⁴⁰The increment for the defendant would never be so great as to make the defendant's next offer equal a sentence greater than the prosecutor's last offer. Likewise, the decrement for the prosecutor would never be so great as to make the prosecutor's next offer equal a sentence less than the defendant's last offer. This is true at any stage in the negotiation process, *see* Section III-B2(c) and Section III-B4, *infra*.

⁴¹By changing the first sign and reversing the order of the limit and the initial offer, an alternative way to write the prosecutor's first counter offer would be $O_{t1} = O_{t0} + RP(ALP - O_{t0})$, which would achieve the same effect although in a less obvious manner. The defendant's first counter offer could be similarly rewritten.

Thus, the first counter offer for the defendant can be defined as $DO_{t1} = O_{t0} + RD(ALD - O_{t0})$, and the first counter offer for the prosecutor as $PO_{t1} = O_{t0} - RP(O_{t0} - ALP)$. The product of the splitting rate and the distance to be covered is preceded by a plus sign for the defendant since it is a positive increment. A minus sign is used for the prosecutor, because his is a negative decrement. DO_{t0} is subtracted from ALD in determining the distance left to be covered by the defendant, because ALD is always larger than DO_{t0} , but ALP is subtracted from PO_{t0} in determining the distance to be covered by the prosecutor, because PO_{t0} is always larger than ALP.

⁴²*See* notes 29-37, *supra*, & text accompanying.

.5. Perhaps they should have had closer splitting rates since their exaggeration factors are the reciprocals of each other. However, since the hypothetical defendant is more anxious to avoid trial, perhaps he should have a higher splitting rate than the prosecutor. On the other hand, given the greater financial pressures on the prosecutor, perhaps he should have a higher splitting rate, since empirical reality may indicate that prosecutors are generally more willing to settle out of court than are defendants. Since good arguments can thus be made for different relative rankings of the RD and RP splitting rates, the RD of .3 and the RP of .5 will be assumed to be reasonable. Over the time points in the dynamic bargaining process, either RD or RP can vary as a result of changes in one's willingness to go to trial or one's bluffing psychology or the reactions of the opponent. For the sake of simplicity, however, FIGURE 2 shows RD and RP as being constant across the graph.

Applying the formulas for calculating the first counter offers of the defendant and the prosecutor to the data provided in FIGURE 2, the defendant's first counter offer is predicted to be 3.58 years or about $3\frac{1}{2}$ years. Likewise, the prosecutor's first counter offer is determined to be 6.51 years or about $6\frac{1}{2}$ years. The calculations are shown at the bottom of FIGURE 2. Thus, at the first stage, the bargainers have not yet converged on a common settlement sentence. In fact, neither side has yet crossed the limit of the other side. Before proceeding to the next stage, it seems appropriate to briefly develop some formulas that have greater generality and insight value in calculating counter offers.

(b) Incremental counter offer

From the above definitional equations of the first counter offers, a general incremental equation determining the counter offer at any stage can be derived. That general equation is $O_{ii} = O_{ii-1} + R(L - O_{ii-1})$. In this equation, O_{ii} is the counter offer at time i ; O_{ii-1} is the counter offer of the previous stage or time i minus one time unit; R is the splitting rate for either the defendant or the prosecutor; and L is the bargaining limit for either party. Since L for the defendant is greater than any of his counter offers, then $L - O_{ii-1}$ will be positive, and R times that distance will be a positive increment for the defendant. Since L for the prosecutor is less than any of his counter offers, then $L - O_{ii-1}$ will be negative, and R times that distance will be a negative decrement for the prosecutor.

With the general counter offer equation we can derive a number of other useful definitions and equations. The absolute difference between $L - O_{ii-1}$ is the distance left to be covered after stage O_{ii-1} . The product $R(L - O_{ii-1})$ is the positive or negative increment to be added to O_{ii-1} to get O_{ii} . The sum of all the increments from the initial offer to O_{ii} can be added to O_{i0} to get O_{ii} . If O_{ii} is the infinite stage, then the sum of all the increments added to O_{i0} will equal the bargaining limit toward which the offers are moving. Since each increment represents only a portion of the remaining distance,

there is never an increment that completes the remaining distance to the bargaining limit unless the bargainer changes his splitting rate or his increment. At any point in the process, a bargainer can change his splitting rate or his bargaining limit if he acquires new perceptions of PC, the payoff cells, or new perceptions of how far he can push the other side.

Applying this general formula for determining counter offers, we find that the second counter offer for the defendant equals $O_{2,1} = RD(ALD - O_{2,1})$, or 4.16, *i.e.* $3.58 + .3(5.5 - 3.58)$. Similarly, the second counter offer for the prosecutor equals $O_{2,1} + RP(ALP - O_{2,1})$, or 5.43, *i.e.* $6.51 - .5(6.51 - 4.34)$. The defendant's second counter offer of 4.16 years still has not crossed above the prosecutor's lower limit of 4.34 years, even though the prosecutor's second counter offer of 5.43 years has crossed under the defendant's upper limit of 5.50 years. This means the prosecutor may be getting a little frustrated, not because he has crossed the defendant's upper limit, which he is not likely to know, but rather because the defendant has still not crossed the prosecutor's lower limit. The defendant, on the other hand, may be gloating because he is succeeding in getting a good bargain, but like a good poker player he will push for an even bigger pot if he can get it.

(c) General solution counter offer

One problem with the general equation for determining the counter offer at any given stage is that it requires knowing the counter offer at the previous stage, which requires knowing the counter offer at the stage before that, and back to the initial offer. That was no problem when the first counter offer was calculated. At later counter offers, though, a more useful equation should state the value of O_{ii} (or the counter offer at any stage or time i) in terms of the initial offer, the limit, and the splitting rate without requiring knowledge of the previous counter offers. This involves solving for O_{ii} in terms of O_{i0} , L , and R with no O_{i-1} on the right side of the equation. Applying a more complex mathematical technique, the general solution is $O_{ii} = L + (1 - R)^i (O_{i0} - L)$.⁴³

Applying the new solution equation to the third stage, the third counter offer of the defendant equals $ALD + (1 - RD)^i (O_{i0} - ALD)$, or 4.56, *i.e.* $5.50 + (1 - .3)^3 (2.75 - 5.50)$. That is exactly the same result determined by the definitional equation, calculating each successive counter offer until the counter offer for the third stage is found. Similarly, the third counter offer for the prosecutor equals $ALP + (1 - RP)^i (O_{i0} - ALP)$, or 4.88, *i.e.* $4.34 + (1 - .5)^3 (8.68 - 4.34)$. Note that the solution equation not only saves times,

⁴³The general definitional or incremental equation cannot be solved for O_{ii} by simple high school algebra. Instead, the methods developed in slightly more advanced algebra for solving difference equations must be used. A difference equation is an equation that involves an expression (like O_{ii}) which is a function of itself at an earlier or later integer point in time (like O_{i-1}) plus or minus an increment, as in the equation $O_{ii} = O_{i-1} + R(L - O_{i-1})$. The solution of this equation involves using a combination of the rules of difference equations, high school algebra, and geometric progressions, and is $O_{ii} = L + (1 - R)^i (O_{i0} - L)$.

but also avoids clerical and rounding errors which can occur from successive calculations with the incremental or definitional equation.⁴⁴

4. Convergence: Its Occurrence, Meaning, and Likelihood

At the third stage, the prosecutor *and* the defendant are within each other's bargaining limits. This means there will be convergence. The parties would be likely to converge from the very beginning, since the defendant's limit is higher than the prosecutor's limit; but coming within each other's bargaining limits makes convergence more nearly certain. However, just because both parties are within each other's bargaining limits does not mean the counter offers are completed. If counter offers at the next stage do not cross over each other, then another set of counter offers will be made. In FIGURE 2, however, at the fourth stage the defendant's counter offer would be 4.84 or about 5 years, and the

The solution is as follows:

EQUATION	SOURCE
1. $O_{ti} = O_{ti-1} + R(L - O_{ti-1})$	(Our basic definitional equation)
2. $O_{ti+1} = O_{ti} + R(L - O_{ti})$	(Eq. 1 applied to stage $ti+1$)
3. $O_{ti+1} = O_{ti} + RL - RO_{ti}$	(Multiplying by R to remove the parentheses)
4. $O_{ti+1} = O_{ti}(1 - R) + RL$	(Factoring out the O_{ti} so it appears only once on the right side)
5. $O_{t1} = O_{t0}(1 - R) + RL$	(Eq. 4 applied to stage $t1$)
6. $O_{t2} = [O_{t0}(1 - R) + RL](1 - R) + RL$	(A combination of Eq. 4 and Eq. 5 applied to stage $t2$)
7. $O_{ti} = O_{t0}(1 - R)^{ti} + RL(1 - R)^{ti-1} + \dots + RL(1 - R)^{ti-ti}$	(Simplifying and removing brackets from Eq. 6)
8. $O_{ti} = O_{t0}(1 - R)^{ti} + L - L(1 - R)^{ti}$	(Inserting and simplifying the sum of a geometric progression for the $+\dots+$ expression in Eq. 7)
9. $O_{ti} = L + (1 - R)^{ti}(O_{t0} - L)$	(Simplifying Eq. 8)

Note the sum of the geometric progression (e.g., $.2 + .4 + .8 + .16 + .32$, etc.) in Eq. 7 is $[RL - (RL)(1 - R)^{ti}] / [1 - (1 - R)]$.

The meaning of the above solution equation is fairly simple given the equations previously explained. To find O_{ti} in the definitional equation, add an increment for the defendant or subtract a decrement for the prosecutor from their previous counter offers. To find O_{ti} in the above solution equation, subtract from the defendant's bargaining limit or add to the prosecutor's bargaining limit. In the solution equation, $O_{t0} - L$ is the distance to be covered from the initial offer to the bargaining limit. It is a negative number for the defendant and a positive number for the prosecutor. The $1 - R$ raised to the " ti " exponent indicates that a smaller portion of that total distance to be covered is added as a positive or negative increment to L at each successive ti stage. The $1 - R$ will always be a decimal smaller than 1, and when a decimal smaller than 1 (unlike an integer) is raised to the exponent 3 (the third time stage) the resulting decimal is smaller than when the decimal is raised to the exponent 2 (the second time stage). For example, $.7$ squared is $.49$, but $.7$ cubed is only $.34$. The reason the expression $1 - R$ is used rather than just R is because we are going backward from the bargaining limit to O_{ti} rather than going forward from O_{ti-1} to O_{ti} .

For further detail on solving difference equations, see BRENNAN, PREFACE TO *ECONOMICS* 71-74, 238-40 (1973); C. DINWIDDY, *ELEMENTARY MATHEMATICS FOR ECONOMISTS* 199-216 (1967). For further detail on finding the sum of a geometric progression, see G. MOORE, *ALGEBRA* 138-44 (1952).

⁴⁴A possibly easier although normally less accurate and less informative way of arriving at a general solution counter offer equation is to use the incremental or definitional equation to calculate two counter offers along with the initial offer. The values of A and B are

prosecutor's counter offer would be 4.61 or about $4\frac{1}{2}$ years. It is impossible or at least very unlikely for the prosecutor to ask the defendant to go to jail for $4\frac{1}{2}$ years when the defendant wants to go to jail for 5 years, or for the defendant to want to go to jail for 5 years when the prosecutor is only asking for $4\frac{1}{2}$ years. Therefore, the fourth stage is an unrealistic occurrence, and convergence is likely to be at the midpoint (4.72) between the two third stage counter offers of 4.88 and 4.56.⁴⁵

Using the static equilibrium approach, the settlement point was the midpoint between the bargaining limits of the defendant (5.50) and the prosecutor (4.34), namely 4.92. Which convergence point makes more sense, the 4.72 or the 4.92? The 4.72 arrived at through the dynamic-equilibrium, difference-equation approach makes more sense because it uses more information, assuming, of course, that this information has some degree of accuracy and is not mere random numbers. To arrive at a settlement sentence, the dynamic equilibrium approach uses the bargaining limits, the only information used in the static equilibrium approach, plus EF, R, and the difference equations.

In discussing convergence under the static equilibrium model⁴⁶ and with regard to initial offers,⁴⁷ the discussion emphasized that when ALD is greater than ALP, the defendant and the prosecutor should be able to agree somewhere between those two limits, since they will then both be coming out ahead of their fall-back positions. However, even if ALD is greater than ALP, strong and persistent bluffing could lead to a break-off in negotiations in which both sides sacrifice the opportunity for a mutually beneficial agreement.⁴⁸ This is more likely to happen if the negotiations involve considerable emotion and name-calling, where a potential agreement collapses because the parties feel that agreement with the other side under such emotional circumstances involves losing face. This fear provides further justification for encouraging the adoption of discovery procedures,

calculated in the regression equation $O_{ti} = A(T+1)^B$, where O_{ti} is the counter offer at stage i , and T is the stage number. For example, to fit such a power function to the defendant's ascending staircase in FIGURE 2, the logarithms of 2.75, 3.58, and 4.16 as the dependent variable scores and the logarithms of 1, 2, and 3 as the independent variable scores are fed into the linear regression analysis. The computer output then informs us that the power function which best fits the defendant's ascending staircase is $O_{ti} = 2.75(T+1)^{.38}$. This equation tells us that when $T = 0$, the initial offer will be 2.75; when $T = 1$, the first counter offer will be 3.58; when $T = 2$, the second counter offer = 4.17; and when $T = 3$, the third counter offer = 4.66. This third counter offer is slightly higher than the 4.56 calculated the more accurate way. Applying the same method to the prosecutor's descending staircase we get the equation $O_{pi} = 8.70(T+1)^{-.43}$. This approach to solving a difference equation is especially helpful if one wants to know what the counter offer is at some time stage between the integer time stages as might be the case in some other substantive problem. What we in effect are then doing is approximating a solution for a differential equation which is generally even more difficult to solve than a difference equation where T must always take integer values.

⁴⁵See Section III-B4, *infra*, for a discussion of the logic behind finding convergence at the midpoint between the last two counter offers before the parties cross over.

⁴⁶See Section III-A1, *supra*.

⁴⁷See Section III-B2(c), *supra*.

⁴⁸See Section III-B2(a), *supra*.

which will enable both sides to have a greater awareness of what the conviction probabilities and sentencing payoffs are likely to be, and to thereby decrease the distrust and short-sighted emotion which may otherwise preclude effective negotiations.

Who won the plea bargaining contest? In a sense, both sides won. The defendant won, since he was willing to go to jail for as long as 5.50 years, but plea bargained a sentence of only 4.72 years. The prosecutor also won since he was willing to let the defendant off with only 4.34 years, but instead obtained a longer sentence of 4.72 years.⁴⁹ In a way, the defendant won more, since the difference between 5.50 and 4.72 (.78) is larger than the .38 difference between 4.72 and 4.34. On the other hand, the extent to which the defendant or prosecutor won should probably be judged by limits the defendant and prosecutor would have had if they had been omniscient beings accurately perceiving PC and the payoffs cells. Judged by that standard, the defendant may have been cheated because he set his limit too high, or he may have enjoyed an unperceived windfall if he set his limit too low. This is also true for the prosecutor. In another sense, the prosecutor could be said to have won more than the defendant since he did obtain a conviction. In that sense, the prosecutor won 4.72 years, and the defendant lost 4.72 years, although that interpretation fails to recognize possible opportunity costs to both parties with regard to what they could have had. Who won thus depends on whether one talks in terms of a victory over what could have been or whether one talks simply in all or nothing terms of conviction or acquittal. The problem is largely semantic and psychological and is really not so relevant to analyzing the process of plea bargaining.⁵⁰

Rather than try to analyze who won or will win in a plea bargaining situation, it is more meaningful to analyze or predict whether or not a plea bargaining settlement will be reached. The circumstances that lead to static equilibrium or disequilibrium have already been discussed in terms of the static model variables of payoff perceptions, probability conviction perceptions, and special types of defendants and prosecutors.⁵¹ The subsequent dynamic equilibrium model increases the understanding of when convergence will be reached by emphasizing the role of bluffing strategies, partly manifested in exaggeration factors, and the role of gullibility or psycho-

⁴⁹If the defendant and the prosecutor are both considered to have won something, then a Pareto optimum solution is obtained, since both sides won relative to their limits, or more narrowly defined, no one lost relative to his limits and at least one party won relative to his limits. If plea bargaining tends to produce such Pareto optimum solutions, it can be considered a Pareto optimizing process.

⁵⁰The discussion focuses upon who won or lost when a plea bargaining settlement is reached. The interesting question of who is likely to win and by how much if the case goes to trial and sentencing is not discussed. See Part I, *supra* note 1, Section II-B3 (on determining the conviction probabilities of who is likely to win) & Section II-A2 (on determining the payoff sentences).

⁵¹See Section III-A, *supra*.

logical willingness to make big jumps in one's counter offers. As in poker playing, these kinds of factors can cause a defendant or a prosecutor to break off negotiations the way a poker player drops out of a hand, even though convergence could have been reached given the respective bargaining limits of the parties.

The dynamic equilibrium material is also valuable for analyzing the speed at which convergence will be reached since the speed of the process is largely dependent on the exaggeration factors and splitting rates of the parties. Convergence speed is also partly dependent on the proximity of the parties' limits which are in turn influenced by their perceptions of PC and the cell payoffs, not to mention their bonus and discount factors. Nevertheless, if the limit of the defendant is only slightly higher than that of the prosecutor, the parties will still not converge quickly if one or both of them is prone to exaggerate or is very parsimonious about splitting the difference between his last offer and his limit. This is especially the case if the parties lack accurate information about the payoffs, the conviction probabilities, and each others' likely limits. With more accurate information, gross exaggeration and unreasonable parsimony is less likely to be pursued and less likely to waste the time of the bargainers and the criminal justice system.

IV. CIVIL ANALOGIES AND IMPLICATIONS FOR POLICY AND RESEARCH

A. *Out-Of-Court Civil Settlements*

One useful way to review the mathematical model just presented for handling the plea bargaining process is to apply it to the simpler situation of out-of-court settlements in civil cases. Analyzing civil settlements and the related literature may also help generate insights regarding policy variables which by analogy could influence the likelihood of settlement in criminal cases. Civil settlements are easier to analyze mainly for two reasons. First, virtually all the goals of both the defendant and the plaintiff can be measured in terms of dollars. Dollars represent a common unit that can be added and subtracted even for different kinds of costs and benefits. Working with dollars provides a more precise scale on which to position the costs and benefits, and an easier method for obtaining meaningful data. A second major reason why civil settlements are easier to analyze is that there are only two sellers from which the defendant-buyer can buy or deal with in a civil case. The defendant can either negotiate with the plaintiff or go to trial. Pleading guilty before a judge is not a meaningful alternative for the defendant in a civil case. If a civil defendant pleads guilty before a judge, then the judge in a civil case can only find the defendant liable and assess the damages for which the plaintiff asks. Only through (1) a bench or

jury trial or (2) an out-of-court settlement can a damage figure be reached other than the figure stated in the plaintiff's complaint.⁵²

1. From the Payoff Matrices to the Dynamic Equilibrium

In light of these two simplifying considerations, both sides are, in effect, working with payoff matrices that only have a bottom or trial row, *i.e.* that have only cells *c* and *d* present. Cell *c* represents the dollar amount that the plaintiff is likely to receive if he goes to trial and does not win a judgment in his favor, *i.e.* when $PL = 0$ where PL is the probability of liability being established. Thus, cell *c* is zero dollars. Cell *d* represents the dollar amount that the plaintiff is likely to receive if he goes to trial and wins a judgment in his favor, *i.e.* when $PL = 1.0$. Cell *d* can be determined using methods similar to those used in calculating cell *d* in the plea bargaining situation.⁵³ Thus, the payoff matrices in the civil situation reduce to one row, because there is only one alternative to negotiation. Indeed, the matrices reduce to just one cell, because one of the two cells in that row will always be zero.

Given these simplified payoff matrices or simply the payoff amount perceived by the defendant and the payoff amount perceived by the plaintiff, a strategies graph like FIGURE 1 can easily be constructed. It consists of only two lines. Both are trial lines starting at the zero origin. The defendant's trial line slopes upward toward his perception of the payoff amount, and the plaintiff's trial line slopes upward toward his perception of the payoff amount. The horizontal axis of the figure shows the perceived probability that liability will be established, *i.e.* PL . Thus, the defendant's unadjusted maximum bargaining limit is the point on his trial line at which he perceives PL , and the plaintiff's unadjusted minimum bargaining limit is the point on his trial line at which he perceives PL . Good estimates of PL can be determined using methods like those discussed for determining PC .⁵⁴ Regardless of what PL is determined to be, the unadjusted limit of a party is calculated by multiplying his perceived PL by his perception of the payoff amount.⁵⁵

To adjust the defendant's limit (LD), add a bonus factor. The bonus factor (XD) consists of litigation costs (analogous to the criminal defen-

⁵²For further discussion of the application of mathematical modeling to out-of-court civil settlements, see Note, *An Analysis of Settlement*, 22 STAN. L. REV. 67 (1969); Johnston & Tersine, *The Mathematical Evaluation of Trial versus Settlement*, CASE & COMMENT 3 (Jan. 1973) [hereinafter cited as Johnston & Tersine]; Posner, *An Economic Approach to Legal Procedure and Judicial Administration*, 2 J. LEGAL STUDIES 399 (1973) [hereinafter cited as Posner]. For a good non-mathematical approach to civil negotiation, see H. ROSS, SETTLED OUT OF COURT (1970).

⁵³See Part I, *supra* note 1, Section II-A2.

⁵⁴See Part I, *supra* note 1, Section II-B3(a).

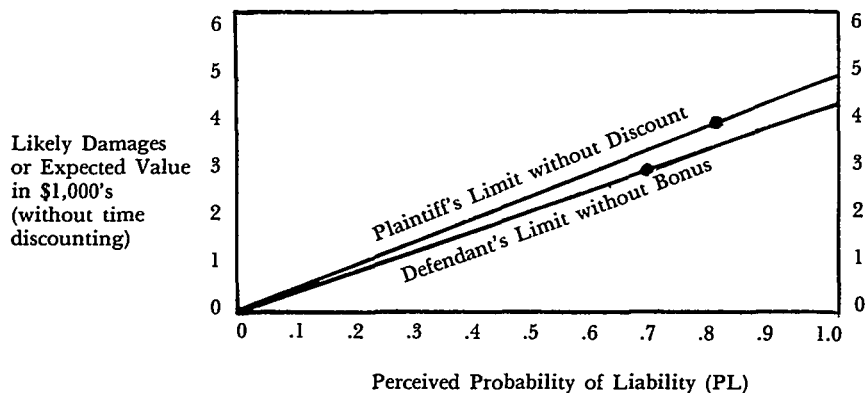
⁵⁵A strategies graph would look like the following for the unadjusted plaintiff's limits and the unadjusted defendant's limits, using the hypothetical data of Section IV-A2, *infra*.

dant's non-sentence goals which tend to turn him away from *litigating*) minus settlement costs (analogous to the criminal defendant's non-sentence goals which tend to turn him away from *settling* through plea bargaining). Thus, as in the criminal situation, ALD or the adjusted limit of the defendant equals $LD + XD$, but here XD nicely equals $\$L - \S , where $\$L$ is litigation costs, and $\$S$ is settlement costs. To adjust the plaintiff's limit (LP), subtract a discount factor. As with the defendant, the discount factor (XP) consists of litigation costs (analogous to the prosecutor's non-sentence goals which tend to turn him away from litigating) minus settlement costs (analogous to the prosecutor's non-sentence goals which tend to turn him away from settling through plea bargaining). Thus, ALP or the adjusted limit of the plaintiff equals $LP - XP$, where XP equals $\$L - \S of the plaintiff.

As in the criminal situation, convergence is likely to be reached if and only if ALD is greater than or equal to ALP. If, on the other hand, ALD is less than ALP, then they will go to trial as the only alternative available, unless at least one of the parties changes his adjusted limits.

The special equilibrium conditions shown in TABLE 2 with regard to certain types of criminal defendants and prosecutors do not apply in the civil case. This is so since there is only one alternative to negotiation, namely going to trial. Thus, civil defendants and plaintiffs cannot be divided into those who see alternative 1, alternative 2, or both alternatives. However, the civil defendants and plaintiffs can be divided on the basis of their degree of knowledge for determining the probability of liability being established under the categories for certainty, ignorance, and risk. That categorization, though, merely indicates that the party to which it applies sees PL as being 0.0 or 1.0 if he is operating under certainty, as being .5 if he is operating under ignorance with middling optimism-pessimism, or as being something between 0.0 and 1.0 if he is operating under risk.⁵⁶

The dynamic equilibrium model is virtually the same for criminal or



⁵⁶See *part I, supra* note 1, Section II-B3.

civil cases. As in criminal cases, the initial offer of the defendant represents his adjusted limit minus an exaggeration factor. Similarly, the initial offer of the plaintiff represents his adjusted limit plus an exaggeration factor. From his low initial offer, the defendant proceeds upward by splitting the difference between his last offer and his relatively high adjusted limit. Likewise, from his high initial offer, the plaintiff proceeds downward by splitting the difference between his last offer and his relatively low adjusted limit. Once both sides come within the adjusted limits of the other side, convergence is virtually assured.

2. Time Discounting

The only complicating aspect of the civil model is the fact that the monetary payoff amounts are generally received substantially into the future. Therefore, they need to be discounted for the passage of time. In other words, assume the average civil personal injury case in a big city like Chicago takes five years to go to trial and to judgment. If the plaintiff is offered \$2,500 now, should he accept it? If he perceives PL to be .80 and the judgment to be \$5,000, then the \$5,000 has an expected monetary value, analogous to a likely or expected sentence in criminal cases, of \$4,000 (.80 times \$5,000). If the defendant's litigation costs are 40 percent of the expected judgment of \$4,000, and his settlement costs are 25 percent of the judgment,⁵⁷ then $\$L - \S is $\$1,600 - \$1,000$. This means the adjusted expected monetary value is \$3,400, *i.e.* 4,000 - 600. In other words, the plaintiff's lower bargaining limit is \$3,400. Without regard to time discounting, he would thus reject the defendant's offer of \$2,500.

The time discounting question, in effect, asks whether the \$2,500 offer is worth more or less than \$3,400 expected value five years hence. It would be helpful to know how much principal would have to be set aside at the current annual rate of interest in order to have in today's money \$3,400 five years from now. Assuming the current interest rate is 6 percent and the inflation rate is 4 percent, then the amount of principal is equal to \$3,080, *i.e.* $\$3,400 / (1 + (.06 - .04))^5$ or algebraically $P = A / (1 + (r - i))^t$, where A is the future value or amount of the investment, r is the interest rate, i is the inflation rate, and t , for time, is the number of years. In other words, the plaintiff would have to be offered \$3,080 now to be receiving the equal of \$3,400 five years from now. Since he is only being offered \$2,500, he should reject the offer.

Time discounting is also important to the defendant. For example, if

⁵⁷The main settlement cost is the one-third contingency fee that usually has to be paid to one's attorney. It represents 33 percent of the settlement. If the expected judgment is higher than the settlement, then 25 percent of the expected judgment might be about equal to 33 percent of the settlement. One often settles for less than the expected judgment in order to save litigation costs which include: (1) the cost of having one's money tied up for a substantial period of time, and (2) the higher contingency fee percentage which personal injury lawyers often charge for going to trial rather than negotiating an out-of-court settlement.

the defendant is made a final settlement offer by the plaintiff of \$3,080 the defendant must decide whether paying the plaintiff \$3,080 now will be cheaper than paying the plaintiff \$3,900 five years from now, assuming \$3,900 is the adjusted upper limit of the defendant.⁵⁸ To determine which is the better deal for the defendant, he must decide what is the value five years hence of \$3,080 paid out now. Assuming a six percent interest rate and a four percent inflation rate, the future value for an investment of \$3,080 in principal is equal to 3,400, *i.e.* $3,080 (1 + .02)^5$ or algebraically $A = P(1 + r)^t$. In other words, the plaintiff's request for \$3,080 now is the equivalent to paying the plaintiff \$3,400 five years from now. Since the defendant has an adjusted upper limit of \$3,900, the \$3,400 figure is a good bargain for the defendant.⁵⁹

The notion of time discounting does not apply to criminal cases in the same way it does to civil cases because there is a much shorter passage of time between the settlement negotiations and the subsequent trial (if there is a trial) in a criminal than in a civil case. Civil cases also deal with money damages (or injunctions against business practices which can be translated into dollars) rather than jail sentences, and the future value of money compared to the present value can be more meaningfully measured than the future and present value of jail sentences. The criminal defendant's lowered bonus factor⁶⁰ does take into consideration that a defendant might be pushed toward going to trial and possibly appeal because he would rather serve five years starting three years from now than four years starting today. Similarly, the prosecutor may offer a bigger discount factor because he would rather put the defendant in jail now, even if it is only a four-year sentence, than put him in jail three years from now after a trial and possible appeal even if the trial yields a five-year sentence. Thus, even in

⁵⁸The \$3,900 ALD was arrived at by assuming the hypothetical defendant perceives the payoff amount to be \$4,500 if the case goes to trial. He also perceives PL to be .70, and thus unadjusted limit is \$3,150. We further assume his litigation costs if the case goes to trial will be \$1,200, and his settlement costs if the case is settled out of court will be \$450. Thus, his bonus factor is .75, and his adjusted limit is \$3,900.

⁵⁹For further detail on time discounting, see C. DINWIDDY, *ELEMENTARY MATHEMATICS FOR ECONOMISTS* 199-216 (1967); C. BELL & L. ADAMS, *MATHEMATICS OF FINANCE* 247-68 (1949). The plaintiff and defendant need not operate under the same discount rate since they may be able to obtain interest on their respective investments at different rates, and they may perceive the inflation rate differently. Similarly, they need not perceive the interest rate or inflation rate as holding constant in each future year. If X changes from year to year (where $X = r - i$), then the formula for calculating P is $P = A / (1 + X_1) (1 + X_2) \dots (1 + X_n)$, and the formula for calculating A is $A = P (1 + X_1) (1 + X_2) \dots (1 + X_n)$ where the subscripts indicate the first year, the second year, and so on. If the defendant were to offer the plaintiff a certain or varying amount of income each year rather than a lump sum, the discounted present value would be calculated by summing the quotient of each year's income divided by $(1 + (r - i))^{t-1}$, where t is one for the first year, two for the second year, and so on. Such an offer is unlikely to be made by the average defendant, but it is a common situation in benefit-cost analysis in deciding between two or more social income producing governmental investments. See E. MISHAN, *ECONOMICS FOR SOCIAL DECISIONS* 112-40 (1972).

⁶⁰See Part I, *supra* note 1, Section II-B4.

the matter of time discounting, the criminal plea bargaining situation involves more considerations which are difficult to measure and more complexities than the civil negotiation situation. Nevertheless, worthwhile insights can be acquired by applying elementary mathematical modeling to both situations.

3. The Decision to Sue and Other Analogies

Before getting to the plea bargaining stage, the prosecutor must decide that he is going to bring a formal charge against the defendant. In a civil case, that decision is analogous to the plaintiff's lawyer deciding that he is going to file a formal complaint. It might be interesting to analyze briefly the general considerations involved in deciding to initiate a civil lawsuit as a grounds for better understanding the analogous considerations involved in deciding to initiate a criminal lawsuit and for better understanding the subsequent plea bargaining process.

In the simplest terms, a plaintiff should initiate a lawsuit if the expected benefits of doing so will exceed the expected costs. The expected benefits to the plaintiff equal the predicted damage award (\$D) times the probability of establishing liability (PL) minus one-third of \$D times PL to cover the lawyer's usual contingency fee. The expected benefits to the plaintiff's lawyer equal one-third of \$D times PL. The expected costs to the plaintiff or the plaintiff's lawyer equal the amount of hours each one is likely to have to spend pursuing the case multiplied by the value of their time per hour, plus the out-of-pocket costs or other non-monetary expenses involved in pursuing the case. The expected costs also include the cost of the missed opportunities to do other more profitable things with one's time where they cannot be simultaneously done.⁶¹ Predicting damages that will be awarded is analogous to predicting the trial sentence in criminal cases.⁶² Determining the probability of liability is analogous to determining the probability of conviction in criminal cases.⁶³

Applying these concepts to the prosecutor, one could say that a prosecutor should initiate a lawsuit if the expected benefits of doing so will exceed the expected costs. His expected benefits equal his predicted sentence multiplied by the probability of obtaining that sentence either through trial, a guilty plea without a bargain, or a guilty plea with a bargain. A prosecutor generally prefers longer sentences to shorter sentences, since a longer sentence tends to indicate that he has caught a more dangerous

⁶¹For analyzing the decision to sue in civil cases, see R. HUNTING & G. NEUWIRTH, *WHO SUES IN NEW YORK CITY?: A STUDY OF AUTOMOBILE ACCIDENT CLAIMS* (1962); F. MAC KINNON, *CONTINGENT FEES FOR LEGAL SERVICES* (1964); Nagel, *Statistical Prediction of Verdicts and Awards*, in *MODERN USES OF LOGIC IN LAW* 135 (Sept. 1963); Schwartz & Mitchell, *An Economic Analysis of the Contingent Fee in Personal Injury Litigation*, 22 *STAN. L. REV.* 1125 (1970). See also note 65, *infra*.

⁶²See Part I, *supra* note 1, Section II-A2.

⁶³See Part I, *supra* note 1, Section II-B3.

criminal, and since the public seems to prefer longer sentences. Like the plaintiff's lawyer in personal injury cases, the prosecutor includes, in his expected costs, his time, expenses, and competing cases that cannot be diverted. Unlike the plaintiff's lawyer, however, the prosecutor may be operating under other constraints. For political and legal reasons, he often must prosecute or formally charge many cases that are not profitable in a benefit-cost sense, although he need not try the case since he still has considerable bargaining discretion. Private defense counsel is more like a private personal injury lawyer in being able to choose his clients. The public defender is more like an insurance company defense counsel since he is required to take all legally eligible cases, although with discretion in out-of-court bargaining.⁶⁴

There are many other aspects of the decision-making activities of lawyers in civil cases that are applicable by analogy to the decision making activities of lawyers in criminal cases besides the settlement process, time discounting, and the decision to sue. Just as one can gain insights into plea bargaining from civil settlements, one can also gain insights into the civil settlement process from analyzing plea bargaining. Thus, much of the earlier discussion on plea bargaining is applicable by analogy to civil cases although generally in a more simplified form. In addition, the model is also applicable to civil cases by deducing the effects of various judicial process changes on the likelihood and level of settlements being reached.⁶⁵ Similarly, suggestions for empirical research on testing hypotheses and methodological tools concerning plea bargaining could also be applied to research on out-of-court civil settlements.⁶⁶

B. *Practitioner and Policy Implications*

1. *Scholars, Lawyers, and Society*

The main purpose of this article is to present a mathematical model that captures the essence of the plea bargaining process in criminal cases. The model presented should have value to legal scholars interested in why the legal process operates the way it does. As such, the model can generate hypotheses which can be empirically tested with data obtainable from questionnaires, interviews, and court records. The model can also serve to integrate related findings that have already been developed concerning the nature of the legal process.

Although the main purpose is the development of conceptual, methodological, and causal theory, the model is also meant to be useful to

⁶⁴For analyzing the decision to formally charge in criminal cases, see F. MILLER, PROSECUTION (1969); Abrams, *Prosecutorial Charge Decision Systems*, 23 U.C.L.A. L. REV. 1 (1975); McIntyre & Lippman, *Prosecutors and Early Disposition of Felony Cases*, 56 A.B.A.J. 1154 (1970). The discretionary decision of private criminal defense counsel to accept a client is discussed in A. WOOD, CRIMINAL LAWYER 96-101 (1967).

⁶⁵See Section IV-B, *infra*.

⁶⁶See Section IV-C, *infra*.

practicing lawyers and legal policy makers. Practicing lawyers like states attorneys, public defenders, and private defense counsel may find insights in the model that will be useful to them in their plea bargaining activities. In personal injury civil practice, there now exist looseleaf services and mathematical how-to-do-it articles designed to aid plaintiffs lawyers and insurance counsel in their negotiation activities.⁶⁷ Perhaps similar tools will become available in criminal case work.

These looseleaf services may be expensive for some attorneys. It might therefore be useful to determine how much they improve the accuracy of the average practitioner in predicting the percentage of plaintiff victories in one hundred cases and in predicting the mean amount of damages awarded. If such a determination were made, one could then better decide whether the extra accuracy is worth the extra cost. To determine how much the defendant or prosecutor would benefit from the improved knowledge gained from such a looseleaf service, one could calculate the absolute value of $S - S'$, where S is the predicted settlement sentence without the use of those services, and S' is the predicted settlement sentence with the improved knowledge. S equals $(ALD + ALP')/2$, where ALD' is greater than ALP' , and where ALD' and ALP' are the defendant's and prosecutor's limits based on improved knowledge of the payoffs and of PC. These same calculations can be used to help determine the extent to which the improved knowledge increases the likelihood of settlement, as well as the accuracy of individual practitioners.⁶⁸

If practicing lawyers in criminal cases can improve their bargaining techniques through a better understanding of decision theory and equilibrium models, society will receive at least four kinds of benefits. First, improving the effectiveness of the prosecutor is socially beneficial since he represents society in the judicial enforcement of social norms. Second, improving the effectiveness of public defenders and defense counsel is socially beneficial to the extent that effective representation improves the respect for the law of people accused of crimes. Many inmates object more to how their plea bargaining was handled than to any other aspect of the criminal justice process.⁶⁹ Third, improving the plea bargaining of both the prosecutor and the defendant reduces the occurrence of under-

⁶⁷H. LIEBENSON & L. MILLER, MEDICAL AND LEGAL EVALUATION OF DISABILITY IN PERSONAL INJURY CASES (1962); Hermann, *Predicting Verdicts in Personal Injury Cases*, 9 PRAC. LAW. 83 (1963); Johnston & Tersine, *supra* note 36; Reeder, *Formulae for Evaluation of Damages*, ABA LAW NOTES (Jan. 1967). The *Verdict Expectancies* looseleaf service of the Jury Verdict Research Company provides probabilities of liability for a large variety of factual situations. The *Valuation Handbooks* of the Jury Research Company and the *Current Money Awards* of the Lawyers Co-operative Publishing Company provide expected damage award payoffs also for a large variety of situations.

⁶⁸See S. RICHMOND, OPERATIONS RESEARCH FOR MANAGEMENT DECISIONS 540-56 (1968); for a discussion of some of the mathematical considerations involved in determining the incremental value of additional information.

⁶⁹J. CASPER, AMERICAN CRIMINAL JUSTICE 100-25 (1972); Blumberg, *The Practice of Law as Confidence Game*, 2 LAW & SOC. REV. 15 (1967); Stover & Eckart, *The Indigent's Right to*

sentencing and over-sentencing. Both kinds of sentencing are socially undesirable since under-sentencing may decrease the deterrent effect of the law, and over-sentencing may generate unnecessary anti-social bitterness. Both under-sentencing and over-sentencing can thus lead to an increase in crime and crime costs. Fourth, saving time is important in the criminal justice process, and improved plea bargaining may enable the prosecutor and defense counsel to arrive at agreements more quickly.

2. The Role of Discovery, Defense, Bail, and Delay

In addition to the direct practitioner and indirect social benefits, an empirically valid plea bargaining model can also be useful for generating ideas among policy makers with regard to (1) how to facilitate fair out-of-court settlements in criminal cases, and (2) the effects of other policies and changes on the settlement process. If a legislature wants to encourage plea bargaining in order to reduce court congestion, save money, and resolve cases more quickly, the most meaningful policy reform that could be adopted is probably the improvement of the criminal discovery process. The model does indicate that if the defendant and the prosecutor are both knowledgeable as to the true payoff cells and the true conviction probability, then they are quite likely to converge at a settlement point, because (1) their unadjusted bargaining limits will then be equal, (2) the adjusted bargaining limit of the defendant will increase in order to avoid the defendant's litigation costs, and (3) the adjusted bargaining limit of the prosecutor will decrease in order to avoid the prosecutor's litigation costs. Through discovery techniques analogous to those used in civil cases, the defendant and the prosecutor can become more knowledgeable about the available evidence, which is valuable in accurately determining the true conviction probability and the sentencing payoffs.⁷⁰

Counsel (Mimeographed paper presented at the Midwest Political Science Association convention, 1973) [on file at the INDIANA LAW JOURNAL].

⁷⁰These discovery techniques include requiring both sides to provide lists of witnesses in advance, transcripts of confessions, itemized lists of materials obtained through search warrants or other searches, and requiring both sides to answer questionnaire interrogatories subject to the privilege against self-incrimination. Related techniques that have been used in civil cases to make both sides more knowledgeable and thereby facilitate settlements include pretrial conferences and examinations by impartial medical experts and other experts.

On discovery in criminal proceedings, see R. NIMMER, *THE OMNIBUS HEARING: AN EXPERIMENT IN RELIEVING INEFFICIENCY, UNFAIRNESS AND JUDICIAL DELAY* (1971); Symposium, *Pre-trial Discovery in Criminal Proceedings*, 27 BROOKLYN L. REV. 318 (1961); Louisell, *Criminal Discovery: Dilemma Real or Apparent?* 49 CAL. L. REV. 56 (1961). Providing more discovery to both sides does not necessarily mean requiring pretrial conferences or omnibus hearings. It would be wasteful to require discovery conferences in all criminal cases because many would result in a non-negotiated guilty plea without necessitating such a conference, and many might be unlikely to result in a negotiated guilty plea even with such a conference. The only types of cases, if any, for which such conferences should be required are those that have characteristics (possibly revealed at the preliminary hearing stage) that are likely to benefit from such a conference. Relevant characteristics might include the severity of the

The model can aid the policy maker not only in stimulating ways of facilitating settlements, but also in clarifying the impact on the settlement process of policy and non-policy changes in the criminal justice system. Two of the most important criminal justice policy changes in recent years have been the increased availability of provided counsel to the indigent and the increased release of arrested persons pending trial. Both of these reforms have had the effect of reducing litigation costs for defendants. The bonus factor will then be reduced, which in turn decreases the likelihood of settlement. Free counsel clearly reduces litigation costs to the defendant although there are two offsetting considerations. First, free counsel tends to consist of a public defender who is pressed to settle because of his heavy caseload and lack of financial and personal resources. Second, increasingly expensive attorney fees for those who are not eligible for free counsel may be increasing the settlement rate among non-indigent defendants.

Bail reform also reduces litigation costs because one of the defendants' trial costs is the cost of having to remain in jail until trial while he is not released on bond. That can be a high cost in terms of lost income and discomfort that pushes jailed defendants toward settlement, *i.e.* offering a larger bonus factor. That bonus element is no longer important when more defendants are being released pending trial. Pointing out these effects of providing free counsel and more liberal pretrial release is not meant to attack those reforms. On the contrary, the purpose is to indicate that the due process benefits the changes provide are partly offset by their settlement reduction costs and to indicate a possible need for other policies, like improved discovery techniques, to counteract that settlement reduction. It should also be pointed out that although providing effective counsel and pretrial release may decrease the likelihood of reaching a settlement, they may cause the resulting settlement to be closer to the true sentence which would have been given at trial since the defendant's artificially high bonus factor is reduced.⁷¹

An example of a non-policy change in recent years that has probably affected the likelihood of plea bargaining convergence is the increased

initial charge and the initial offers, if any, by the prosecutor and the defendant. If the severity is very low, the conference can be optional since guilty pleas are still likely to be high. If the parties are very close on their initial offers they will probably reach convergence without a compulsory conference; and if they are very far apart on their initial offers, a compulsory conference will not be so likely to bring convergence.

⁷¹See Section IV-B2, *infra*. Providing more counsel to the indigent and more pretrial release decreases ALD, not only by decreasing the defendant's litigation costs, but also possibly by decreasing the defendant's perception of his conviction probability and his sentencing payoffs. Defendants are less likely to be severely convicted when they have a lawyer and when they have been released prior to trial so that they can better prepare their cases. The prosecutor's perception of the conviction probability and the sentencing payoffs may also thereby be decreased. This can result in a new midpoint between ALD and ALP that comes closer to the "correct" sentence if by correct sentence one means that it is not influenced by the discriminatory absence of defense counsel and pretrial release.

delay in processing criminal cases.⁷² This delay is due to increases in population, crime rates, and urbanization although it is partly offset by new court management techniques and new legal rules requiring speedier trials. Long delay from arrest to trial increases the willingness of a defendant to settle if he is held in jail pending trial. On the other hand, it decreases the willingness of a released defendant to settle since a distant trial penalty is less of an incentive to settle than one closer in time. Long delay, however, may increase the prosecutor's willingness to settle since he has a difficult burden of proof which generally becomes harder to meet as witnesses become more forgetful or disappear. In anticipation of that happening, the prosecutor tends to be willing to offer a bigger discount factor. Given these conflicting effects of delay, it is hard to say how increased delay affects cases in general. Specific cases categorized in terms of whether the defendant is in or out of jail and whether the prosecutor is relying on evidence that has a high or low time-decay rate may be better explained.⁷³

3. Changes in Conviction Probabilities and Sentencing Payoffs

What does the model indicate about the effect of increasing or decreasing PC or the cell payoffs on settlement likelihood? Clearly, if the defendant is made to think PC or the cell payoffs are higher than they really are, he will be more willing to settle. Likewise, if the prosecutor is made to think PC or the cell payoffs are lower than they really are, he will also be more willing to settle. As previously mentioned, however, they will both be quite willing to settle if they simply perceive PC and the cell payoffs accurately, rather than falsely upward for the defendant and

⁷²Instead of noting the effect of delay on the likelihood of plea bargaining settlement, one might note that plea bargaining may be one of the causes of delay if continuances are often required to allow plea bargaining to occur and a large percentage of plea bargaining negotiations break down. See Levin, *Delay and Related Policy Topics in Five Criminal Courts*, (mimeographed paper presented at the American Political Science Association convention, 1973) [on file at the INDIANA LAW JOURNAL]. That relation possibly further emphasizes the need to facilitate convergence in plea bargaining by making both sides more knowledgeable through discovery procedures.

Abolishing or restricting plea bargaining would normally increase delay in the criminal justice system by increasing the number of trials. The amount of delay depends on the number of trials, the number of persons available to try cases, and the average length of a trial. One could thus offset the increased delay by increasing the number of persons available to try cases or decreasing the average length of trial. The prosecutor could also offset the increased delay by dismissing more cases. There may, however, be no increased trials or delay if the decrease in plea bargains is offset by an increase in guilty pleas without a bargain out of fear that the likely sentence on trial would be greater than the sentence following a non-negotiated plea. That is what seems to have happened when plea bargaining was abolished in certain types of cases in Phoenix, Arizona. Berger, *The Case Against Plea Bargaining*, 62 A.B.A.J. 621 (1976).

⁷³For further discussion on the role of discovery, defense, bail, and delay in determining settlements, see Landes, *An Economic Analysis of the Courts*, 14 J.L. & ECON. 61 (1971) [hereinafter referred to as Landes]. See also H. KALVEN, JR. & B. BUCHHOLZ, *DELAY IN THE COURT 3* (1959); M. ROSENBERG, *THE PRETRIAL CONFERENCE AND EFFECTIVE JUSTICE* (1964); Posner, *supra* note 52, at 399-458.

downward for the prosecutor, since the defendant's bonus factor and the prosecutor's discount factor will then put the defendant's bargaining limit above that of the prosecutor.

What if the true PC increases or decreases from improved or worsened investigative and presentation abilities by the prosecutor relative to defense counsel, or from changes in the rules of evidence? FIGURE 1 helps answer that question. It reveals that if both parties accurately perceive the same true PC, the defendant's upper limit will be higher than the prosecutor's lower limit without regard to that jointly-perceived true PC. The gap between the defendant's upper limit and the prosecutor's lower limit, however, is wider when PC equals 1.0 than when PC equals 0.0. That is partly due to the relative payoffs of the defendant and prosecutor shown in TABLE 1. If they both had the same payoffs, then they would have the same limit lines, and they would be willing to set the same price at any jointly-perceived low or high PC. If, however, the defendant is willing to give a smaller bonus at a lower PC, then he will be less willing to settle at a lower PC. This, however, may be offset by the fact that the prosecutor is willing to give a higher discount and more willing to settle at a lower PC. Thus, improving or worsening the probability of conviction does not generally affect the likelihood of a settlement being reached except in those situations where the defendant perceives the payoffs as being higher than the prosecutor does, (and settlement will thus be more likely at higher PC's), or where the defendant increases his bonus more for a higher PC than the prosecutor decreases his discount (and settlement will thus also be more likely at higher PC's).

What if the true payoffs increase or decrease from more harsh or lenient sentencing? FIGURE 1 is also helpful in answering the effect of that kind of policy change. If sentencing became more harsh, the limit lines of both the defendant and the prosecutor would still start at the zero origin on the left vertical axis since the defendant would receive no sentence if he goes to trial when his probability of conviction is zero. The limit lines of both parties would, however, be higher on the right vertical axis. For example, those intercept points are now seven and six for the defendant and prosecutor, respectively, but with twenty percent harsher sentences they might become about $8\frac{1}{2}$ and 7, respectively. Nevertheless, the parties would still be as likely to reach a settlement, unless the defendant perceived the increase as being substantially greater than the prosecutor (which would increase the likelihood of settlement), or the defendant perceived the increase as substantially less than the prosecutor (which would decrease the likelihood of settlement). If FIGURE 1 were redrawn so that both parties initially perceived the payoff cells alike and thus had the same limit lines, then an upward or downward change in sentencing practices would simply cause their common limit line to shift up or down without affecting the likelihood of settlement, assuming they both accurately perceive the

sentencing shift. The same results are true if changes in sentencing practices affect only pleading before a judge, going to trial, or one of the four cells.⁷⁴ This, however, assumes that the increase or decrease in sentencing practices does not affect the relative size of the defendant's bonus and the prosecutor's discount. The more severe a case, the less willing the defendant might be to offer a large bonus for a plea bargaining settlement and in effect plead for a long jail sentence; but this is offset by the possible fact that the more severe a case, the more willing the prosecutor might be to offer a large discount to avoid an expensive trial.⁷⁵

⁷⁴Landes, *supra* note 73, at 69-71, however, states that the more severe a case is, the more resources the defendant devotes to fighting the case, which decreases his PC and thereby increases his willingness to go to trial and his unwillingness to agree to a plea bargain settlement. That analysis, however, may fail to recognize that the more severe a case is, the more resources the prosecutor devotes to fighting the case, which may restore the decreased PC to where it was. Even if the prosecutor does not devote additional resources to the more severe case, if he is a perceptive prosecutor, he will perceive that PC has gone down as a result of the defendant's increased resources. The prosecutor's new lower PC will then result in a lowered bargaining limit point for the prosecutor which will still enable a settlement to be reached. This is so since LP will be equal to LD when both sides perceive PC and the payoff cells alike, and ALP will be lower than ALD when the litigation costs are greater than the settlement costs.

⁷⁵See Part I, *supra* note 1, Section II-B4. In Posner, *supra* note 52, at 421, however, the writer says that if the stakes become higher in civil cases, the parties are less likely to settle. The only example he gives is the addition of an interest rate percentage onto the damage award rather than a fixed addition or subtraction. Thus, if in Posner's example, LD = \$100, LP = \$120, then the initial gap is -\$20. If six percent interest is added to each limit, then LD = \$106, LP = \$127, and the new gap is a larger -\$21. If, however, a \$6 penalty had been added to the damage award for delay rather than six percent interest, then LD = \$106, LP = \$126, and the gap is still -\$20.

More important, Posner assumes the perceptions (and thus the limits of the parties) are unequal with LD being unrealistically lower than LP. If LD of \$100 were equal to LP of \$100 as in the perfect discovery situation, then after adding six percent interest, LD = \$106, LP = \$106, and the gap remains the same. If, more realistically, ALD were to be \$120 after the bonus factor, and ALP were to be \$100 after the discount factor, then the six percent interest would make ALD = \$127, ALP = \$106, for a gap of \$21 which is more likely to induce a settlement since it allows more room for convergence than a gap of \$20 when ALD is greater than ALP. In other words, the bigger the gap, the less likely the settlement when ALD is smaller than ALP; but the bigger the gap, the more likely the settlement when ALD is larger than ALP.

Posner also assumes that the likelihood of settlement is dependent only on the size of the gap between the limits of the parties and not on the gap ratio. Clearly, the size of the gap is more important than the gap ratio in that the parties would be more likely to settle in case one than in case two where in case one, LD = \$10, LP = \$11, the gap = -\$1, and the gap ratio = .91 (or \$10/\$11), and where in case two, LD = \$1,000,000, LP = \$1,100,000, the gap = -\$100,000, and the same gap ratio = .91. Nevertheless, if the gap is held roughly constant, the case with the larger gap ratio is more likely to be settled such that case two is more likely to involve a settlement than case three where in case three, LD = \$1, LP = \$100,001, the same gap = -\$100,000, but the gap ratio = .00001. In other words, in case two, the defendant-buyer can come up to 91 percent of the plaintiff-seller's asking price; whereas in case three, the defendant-buyer can only come up to 1/1000 of 1 percent of the plaintiff's asking price even though in both cases they are \$100,000 apart. This problem of the relative importance of the gap and the gap ratio is related to the fundamental benefit-cost decision problem of the importance of maximizing benefits minus costs (B-C) versus maximizing benefits divided by costs (B/C), assuming both benefits and costs can be measured in the same units. Analogous to the gap and gap ratio comparison, B - C is the preferable criterion; but if two investments are equal on B - C, then one should prefer the investment with the higher B/C ratio. The above

The above discussion of the non-effects of increasing PC or increasing the likely sentences on settlement likelihood seems to run contrary to the common sense notion that if PC and the likely sentences are increased, then the defendant should be more willing to settle and avoid a likely conviction carrying a stiff sentence. This common sense notion, however, assumes, perhaps falsely, that the plea bargaining process on the part of the prosecutor and the defendant fails to take into consideration the upward shift in PC or in the likely sentences. In other words, to the extent that the prosecutor and the defendant correctly perceive those upward shifts and rationally incorporate them into their bargaining limits, then the results of the plea bargaining will still be convergence, although at a higher sentence than would otherwise be the case. This means the defendant would be no more likely than before to reach a plea bargaining settlement, since such a settlement will now mean a higher sentence. At the same time, however, he will be no less likely to reach a settlement since the alternative sellers from whom the defendant-buyer can buy have now also raised their prices. The most important meaning of this analysis, though, is that rational plea bargaining can just as capably produce meaningful sentences where the parties accurately perceive PC and the sentencing payoffs as can the trial process, although plea bargaining does so with less expenditure of social resources.

The same kind of conclusion could have been reached by noting that if PC and the likely sentences decrease, then the defendant will be less willing to settle because the prosecutor cannot so effectively threaten him. This conclusion, however, fails to consider that the PC reduction and sentence reduction tends to get incorporated into the plea bargaining on both sides, so that the parties are now logically bargaining between lower limits. This means that the defendant will be no less willing than before to reach a settlement, since such a settlement will now mean a lower sentence. At the same time, he will be no more likely to reach a settlement, since all the alternative sellers from whom the defendant-buyer can buy have now also lowered their prices.

As a result, plea bargaining thus tends to result in sentences that reflect the true probabilities of conviction and the true likely sentences,⁷⁶ *provided that:*

examples could illustrate that principle by changing LP to B and LD to C. See R. McKEAN, *EFFICIENCY IN GOVERNMENT THROUGH SYSTEMS ANALYSIS* 25-49 (1958); E. MISHAN, *ECONOMICS FOR SOCIAL DECISIONS* 134-35 (1972).

⁷⁶One might argue that even if plea bargaining could be made to arrive at sentences that on the average equal those that would be arrived at through trials, the plea bargaining system would still be defective for at least two reasons. First, plea bargaining does not give the appearance of due process which a trial does. Therefore, defendants who are convicted by way of plea bargaining might be more likely to have anti-social resentment. This is currently likely to be so where defendants feel they are coerced into pleading guilty for lack of resources to take cases to trial. If, on the other hand, public defenders have overly adequate resources, defendants might feel they were getting such a break under the system that the system might lose some of its deterrent power. Second, even if the plea bargaining averages equal the trial

1. The parties are as capable as possible of accurately perceiving PC and the sentence payoffs, which can be facilitated by better discovery procedures and more objective sentencing.

2. The defendant is not forced to offer an excessive bonus (or any bonus), which he otherwise might (a) if he were being held in jail pending a distant trial, (b) if he could not afford an expensive lawyer and was not eligible for a free one, or (c) if he has a public defender who does not have the time or resources to take cases to trial where a trial would bring a lower likely sentence than plea bargaining would.

3. The prosecutor is not forced to offer an excessive discount (or any discount), which he might be forced to do if he did not have the time or resources to take cases to trial where a trial would bring a higher likely sentence than plea bargaining would.⁷⁷

averages, it would not be any consolation to tell an individual who receives an above-average sentence in a plea bargain that someone else got an offsetting below-average sentence. What is needed is to strive to have a system whereby every plea bargain, not just the average plea bargain, equals the trial sentence. This means not only adequate resources for the defense and prosecution, but also possibly pretrial discovery and flat sentencing, whereby both sides can come close to predicting with certainty whether or not the defendant will be convicted and with what sentence.

⁷⁷For further discussion on the accuracy and fairness of plea bargaining, see AMERICAN BAR ASSOCIATION PROJECT ON MINIMUM STANDARDS FOR CRIMINAL JUSTICE, STANDARDS RELATING TO PLEAS OF GUILTY (1967); NATIONAL ADVISORY COMMISSION ON CRIMINAL JUSTICE STANDARDS AND GOALS, COURTS 42-65 (1973); D. NEWMAN, CONVICTION: THE DETERMINATION OF GUILT OR INNOCENCE WITHOUT TRIAL 7-52, 231-43 (1966); Gallagher, *Judicial Participation in Plea Bargaining: A Search for New Standards*, 9 HARV. C.R. - C.L. L. REV. 29 (1974); Kuh, *Balancing the Scales of Justice: How to Make Plea Bargaining Work*, in THE NEW LEADER 10-14 (Jan. 7, 1974).

The most practical sentence to arrive at in plea bargaining is for S^* (the settlement sentence) to equal PC' times d' where PC' is the true value of PC which would be 0.0 or 1.0, and where d' is the true value of cell d . The value of PC' and d' cannot be known with certainty before trial except by an omniscient being, but both values can be known after trial if the case goes to trial. The S^* will equal $PC'd'$ if (1) the defendant, the prosecutor, and the pleading judge perceive PC as PC' ; (2) they perceive d as d' ; (3) $a = b = d$, which means the same sentence is given on plea as on trial; and (4) the defendant's bonus factor to be added to LD equals the prosecutor's discount factor to be subtracted from LP. With these four conditions, $LS_1 = c + (d-c)PC = 0 + (d' - 0)PC' = PC'd'$, and $LS_2 = a + (b-a)PC = d' + (d' - d')PC' = d'$. Thus both LD and LP equal d' if $PC' = 1$, and both LD and LP equal 0 if $PC' = 0$. Therefore, given those four conditions, $S^* = (ALD + ALP)/2 = PC'd'$. The most ideal sentence to arrive at in plea bargaining is PG times d'' , where PG is the true probability of actually being guilty rather than just being convicted, and d'' (d double primed) is the sentence which is deserved in light of the crime and the defendant's characteristics (according to an omnibenevolent being, a survey of public opinion, or whatever source one regards as ultimate). The value of $PC'd'$ can be objectively determined even before trial with some statistical accuracy, see Part I, *supra* note 1, Sections II-A2, II-B3(a), but not the value of PGd'' .

The above symbolism is useful for obtaining a better understanding of the four roles which Albert Alschuler perceives the prosecutor as having. Alschuler, *The Prosecutor's Role in Plea Bargaining*, 36 U. CHI. L. REV. 50 (1968) [hereinafter cited as Alschuler]. The four roles include that of (1) the administrator who will settle for anything greater than ALP, where ALP equals LP minus a large discount or %XP to avoid trial; (2) the advocate who will try for a bargain as much greater than LP as possible, where LP equals LS_1 or LS_2 , whichever is lower with no discount to promote administrative convenience; (3) the judge-like prosecutor who strives for a settlement at what he perceives d' to be, or the true empirical value of cell d ;

By way of summary with regard to the effect of judicial system changes on the likelihood of a plea bargaining settlement being reached and at what level, one can say the following in light of the model presented:

1. A change that decreases the defendant's bonus factor (such as increased free counsel or pre-trial release) will lower the defendant's adjusted bargaining limit without affecting the prosecutor's limit. This will narrow the room for settlement and lower the level of the new settlement, assuming that a settlement can still be reached, and that it will still be roughly at the mid-point between the defendant and the prosecutor's limits. The opposite occurs from a change that increases the defendant's bonus factor. A prosecutor who is aware that a change has occurred which decreases the defendant's bonus factor can offset the decreased settlements by making better offers. He might especially want to do that if the decreased settlements add to his court congestion and thereby increase his desire to raise his discount factor.

2. A change that decreases the prosecutor's discount factor (such as more resources to the prosecutor thereby in effect lowering the cost of litigation) will raise the prosecutor's adjusted bargaining limit without directly affecting the defendant's limit (although more resources to the prosecutor may also affect the probability of conviction). This will have the effect of narrowing the room for settlement and the effect of increasing the level of the new settlement if one can still be reached. The opposite occurs from a change that increases the prosecutor's discount factor.

3. A change that improves the ability of one or both sides to predict more accurately the probability of conviction or the sentence upon conviction (such as pretrial discovery proceedings or flat sentencing)⁷⁸ will

and (4) the legislator-like prosecutor who strives for a settlement at what he perceives d' to be, or the righteous normative value of cell d .

⁷⁸Flat sentencing whereby the legislature removes judicial discretion to sentence after conviction increases the predictability of all the cells in the decision theory matrix. Thus, if the statute specifies a 10-year sentence for a given crime, then cells a , b , and d will all have 10's in them, and cell c will continue to have a 0. Predictability may, however, be decreased in some non-typical cases if the statute allows the judge to award probation as an alternative to flat sentence, or if the judge or jury frequently refuse to convict in a trial because they consider the flat sentence out of line with the nature of the circumstances.

An important effect of flat sentencing on plea bargaining would be a shift from bargaining over sentencing recommendations to bargaining over the charge. Prosecutors may not be so willing to reduce the charge if a reduced charge means a definite sentence reduction as it does under flat sentencing, but as it often does not under indeterminate sentencing. Another effect would be that going to trial would always be a more favorable alternative than a non-negotiated plea of guilty given the possibility of an acquittal. This might mean more cases going to trial when plea bargaining breaks down than at present, although the extent to which non-negotiated pleas provide an alternative to plea bargaining must be empirically determined. The effect of practically abolishing non-negotiated pleas might cause the prosecutor to be more willing to make concessions on the charge in order to (1) avoid trials, (2) cover the non-typical case, and (3) relieve himself of the increased burden stemming from more cases being thrown into lengthy plea bargaining because the defendant no longer has the option available to him of a non-negotiated plea followed by a light sentence.

The prosecutor's bargaining position would mainly be strengthened and the defendant's weakened if the new flat sentencing results in higher likely sentences than the old

have the effect of increasing the likelihood of settlements by decreasing misperceptions of their bargaining limits by the respective parties. In the normal case, if both parties accurately perceive the probability of conviction and the sentences and thus have the same nonadjusted limits, then a settlement should be reached when the bonus factor raises the defendant's limit and the discount factor lowers the prosecutor's.

4. A change that increases the probability of conviction (such as more lenient admissibility of police-obtained evidence) or that increases sentencing payoffs in any of the decision matrix cells (such as new mandatory minimum sentences) will have the effect of raising both the defendant's adjusted limit and the prosecutor's adjusted limit (if they both accurately perceive the effects of those judicial system changes on PC and on sentencing) since their respective limits at least partly reflect the product of the perceived PC (or 1-PC) times the sentence associated with each decision-making possibility. The new limits will then still allow as much room for settlement as before, but they will both be higher, thereby resulting in settlement at a higher level.⁷⁹ A change that decreases the probability of conviction or the sentencing payoffs will have the opposite effect.⁸⁰

more indeterminate sentencing, or if the likely sentences are perceived as being higher by defendants who may formerly have wishfully misperceived likely sentences as being lower than they actually turned out to be. If the non-negotiated plea formerly provided a more attractive alternative to the defendant (as it probably would if PC were high), then abolishing it would strengthen and thereby raise the prosecutor's limit and weaken or lower the defendant's limit. Thus, the effects of flat sentencing are more complicated than merely improving the predictability of sentences by decreasing their variation. This is so in view of the fact that the flat sentence level could conceivably be higher, lower, or at the same level as the former sentence generally was or was perceived to be, and in view of the influence of the level of PC on the impact of the sentence change on plea bargaining. In general, flat sentencing provides for sentences that are substantially above the former average sentences, but the flat sentences are subject to being reduced close to the average by reductions of one day for each day of good time served in prison.

⁷⁹Although the model enables one to determine the probable direction of the effects of judicial process changes on the likelihood and level of plea bargaining settlements, one can not determine the exact magnitude of such effects in an individual case because doing so depends on the individual prosecutor, defendant, defense counsel, and case facts. However, the magnitude of the effects on the average prosecutor of a given type, the average defendant of a given type, or the average case of a given type can be discussed, at least after compiling further empirical data. See Section IV-C1, *infra*. One can also talk about certain judicial procedures as being more or less conducive to increasing settlements than other judicial procedures such as statements that there will be more settlements if pretrial detention averages six months than if it averages three months and other things are held constant.

⁸⁰Oaks and Lehman argue that "when the overall rate of conviction at trial goes down, so would the proportion of guilty pleas." D. OAKS & W. LEHMAN, *A CRIMINAL JUSTICE SYSTEM AND THE INDIGENT* 57-58 (1968). Such reasoning seems to assume that a drop in PC only affects the behavior of the defendant who now demands a lower sentence to reflect his lowered bargaining limit. In light of the model and in light of the data presented by Oaks and Lehman which they consider contrary to their above common sense, it seems reasonable to expect that the prosecutor now offers a lower sentence to reflect his also lowered bargaining limit. As a result, the percent of cases settled through plea bargaining should remain about the same unless one side perceives the drop as being greater than the other side although the new average sentences should now be lower. It is an empirical question as to which side, if either, has a greater tendency to perceive a PC drop or a sentencing drop as being bigger although

C. Future Research

This article could conceivably lead to two kinds of future research. One kind would relate to the plea bargaining model. The other kind would involve applying the decision theory and equilibrium modeling concepts and methods to other legal process and related problems.

1. Plea Bargaining Research

Future plea bargaining research can serve at least three useful purposes with regard to the model presented. One purpose is to test the meaningfulness of the measuring instruments suggested in this article. The measuring instruments include the methods for (1) deriving the payoff matrices through questioning knowledgeable persons or analyzing case data;⁸¹ (2) converting the payoff cells into relative or ordinal utility measures;⁸² (3) determining conviction probabilities for specific cases or types of cases;⁸³ (4) obtaining an optimism-pessimism coefficient and applying it to narrowing to a point the estimation range on PC or a payoff cell;⁸⁴ and (5) seeing if and how non-sentence goals can be meaningfully reduced to a percentage against which the defendant's upper sentence bargaining limit or the prosecutor's lower sentence bargaining limit can be multiplied in order to determine the bonus or discount factor.⁸⁵

A second purpose of the testing is to determine the extent to which various hypotheses directly generated by the model are true. These hypotheses relate to the internal workings of the plea bargaining process. They can be tested through depth interviewing or possibly through mailed questionnaires that are carefully structured and directed toward prosecutors, defense attorneys, judges, or defendants in various places.⁸⁶

perhaps there is more of a knowledge lag on the part of less knowledgeable defense counsel than there is on the part of prosecutors, who tend to be more full-time involved in criminal work than the average defense attorney. This differential would explain why when PC went down in the Illinois data, prosecutors made better offers resulting in a higher percentage of bargained guilty pleas. In other words, given the small one-year time lag that Oaks and Lehman looked at, there was insufficient time for the original percentage of settlements to restore itself as a result of defense counsel demanding better offsetting settlements.

⁸¹See Part I, *supra* note 1, Section II-A2.

⁸²See *id.* at Section II-A3.

⁸³Methods to be tested for determining conviction probabilities for specific cases or types of cases might include the averaging of knowledgeable persons and the use of the three-point estimate system. See *id.* at Section II-B3(a).

⁸⁴See *id.* at Section II-B3(c).

⁸⁵See *id.* at Section II-B4.

⁸⁶They include such statements as (1) pleading guilty before a judge produces a lower sentence than being convicted at trial for the same crime, (2) pleading guilty before a judge is used as an alternative to plea bargaining with the prosecutor, (3) defendants and prosecutors have fairly clear notions of what the likely sentences are for going to trial and pleading when the evidence is extremely weak or extremely strong, (4) defendants and prosecutors have fairly clear notions of what the probability of conviction is under various circumstances, (5) defendants perceive the likely sentences and conviction probabilities to be higher or maybe lower than prosecutors do, (6) defendants and prosecutors have a number of other goals besides sentence minimization and maximization respectively, (7) the non-sentence goals pushing toward settlement are stronger than the non-sentence goals pushing toward litigation, (8) the non-sentence goals vary with the severity of the case, the time discounting,

A third purpose for future plea bargaining research relevant to the model is to determine the extent to which various hypotheses indirectly generated by the model are true. These hypotheses relate to the effect of external policies, events, or changes on the plea bargaining process. They require studying plea bargaining in different communities, in the same communities over time, or in different cases using large random or matched samples.⁸⁷

A purpose that cannot be served by empirical research like that listed above is the testing of the basic assumptions of the model. This is so because those assumptions are almost definitional tautologies. The basic assumption in the model is that both defendants and prosecutors want to

and the risk-oriented personalities of the parties, (9) plea bargaining settlements occur in a high percentage of felony cases, (10) defendants and prosecutors exaggerate their initial offers, (11) defendants and prosecutors try to make the other side think the conviction probabilities and the sentencing payoffs are lower or higher respectively than they perceive them to be, (12) who makes the initial offer depends more on local custom than on which side has the greater need to avoid trial, (13) counter offers of the defendant tend to move upward with bigger jumps at first than later, (14) counter offers by the prosecutor tend to move downward with bigger jumps at first than later, and (15) both the defendant and the prosecutor feel they have gained something from a plea bargaining settlement over what they expected to get by going to litigation in terms of sentence or non-sentence goals.

For examples of such survey research to determine average and deviant practices, see Vetri, *Guilty Plea Bargaining: Compromises by Prosecutors to Secure Guilty Pleas*, 112 U. PA. L. REV. 865 (1964); Note, *The Influence of the Defendant's Plea on Judicial Determination of Sentence*, 66 YALE L.J. 204 (1956). In hypothesis 15 as elsewhere in this article, the concept of "litigation" as an alternative to plea bargaining refers to going to trial, pleading before a judge, or both, depending on what litigation alternatives are perceived as available to the defendant.

⁸⁷These hypotheses include statements like (1) the presence of criminal discovery laws facilitate plea bargaining settlements, (2) the increased availability of court-provided counsel and pre-trial release have decreased settlements, (3) increased delay in prosecuting criminal cases increases settlements where evidence likely to decay is involved, but decreases settlements where defendants are not in jail pending trial, and (4) changes in the conviction probabilities or sentencing payoffs do not affect settlement rates.

For examples of such cross-sectional or over-time studies to determine relations between practices and varying policies or environments, see Alschuler, *The Prosecutor's Role in Plea Bargaining*, 36 U. CHI. L. REV. 50 (1968); Landes, *supra* note 73. In addition, see the symposium on the comparative study of prosecutors in 5 THE PROSECUTOR 237 (1969).

Thomas Church and William Morris are developing an intensive study of the before and after effects of the discontinuance of charge reduction plea bargaining in Oakland County, Michigan, in which the sale of narcotics is first charged. Church & Morris, *Charge Reduction Plea Bargaining and the Courts: A Model Based on Quasi-Experimental Data* (unpublished paper prepared at Oakland University, 1975) [on file at the INDIANA LAW JOURNAL]. They plan on working with a probabilistic decision-theory model although like most plea bargaining models, they do not consider the option of pleading guilty before a judge without a negotiated bargain, the determination of the bargaining limits of the parties, the dynamics of convergence, the occurrence of bluffing and other psychological elements, and the difficulties in measuring subjective benefits rather than more objective sentences. Their model like most models, however, will aid in generating hypotheses to test for and in synthesizing the data which is compiled. Such an approach can be contrasted with a massive compilation of raw data that has little theoretical direction for hypothesizing or integrating, such as Miller, Dash, & McDonald, *Plea Bargaining in the United States* (funded LEAA application prepared at Georgetown Law School, 1975) [excerpts on file at the INDIANA LAW JOURNAL].

maximize their satisfaction. This is almost a tautology, since one can define satisfaction as what one receives when he chooses one alternative action over another, given his values and the information he has available at the time he makes the choice. Thus, even masochistic and martyr-prone defendants seek to maximize their satisfaction, although they have an unusual value system. Similarly, defendants who represent themselves when they could have court-provided counsel are also seeking to maximize their satisfaction although they may have a faulty information system about the functions performed by defense counsel. At a less basic level, the model assumes that the satisfaction of defendants will increase if their sentences decrease, and the satisfaction of prosecutors will increase if the sentences they obtain also increase, unless some offsetting non-sentence goals are being achieved by the defendant or prosecutor. This assumption is also practically a tautology, since the concept of non-sentence goals is broad enough to include any unusual reason why a defendant would not receive satisfaction from a lower sentence, or why a prosecutor would not receive satisfaction from a higher sentence.⁸⁸

If defendants and prosecutors are assumed to want to maximize their respective satisfactions, then this is like saying they both are rational, or not irrational, if "rational" simply means choosing the alternative action that gives the most satisfaction given one's values and the information he has available at the time he makes the choice. In this context being "rational" does not necessarily mean being consistent in one's values, informed of relevant information, intelligent in IQ, or even capable of functioning with psychiatric sanity. Laymen, however, often use the term "rational" in these narrower senses which makes the term not as meaningful in decision theory as the phrase "wanting to maximize one's satisfaction."⁸⁹ In spite of the simplicity of our basic assumptions, they are the only goal assumptions on which the essence of the plea bargaining model is based.⁹⁰ The calculation of limits like LD, LP, ALD, and ALP, and the calculation of convergence points like S* are deduced from these axioms or assumptions.

⁸⁸The model does not assume that it is possible for the defendant and the prosecutor to bargain with each other since the model allows in Section II-B2(b), *supra*, for the possibility that the defendant's only alternatives are trial or judicial pleading. On axiomatic theory or the deduction of conclusions from empirical premises, see E. MEEHAN, *CONTEMPORARY POLITICAL THOUGHT* 287-349 (1967); Bailey, *Evaluating Axiomatic Theories*, in *SOCIOLOGICAL METHODOLOGY* 1970 (E. Borgatta ed. 1970); Land, *Formal Theory*, in *SOCIOLOGICAL METHODOLOGY* 1971 (H. Costner ed. 1971).

⁸⁹On rationality, see FRIEDLAND, *INTRODUCTION TO THE CONCEPT OF RATIONALITY IN POLITICAL SCIENCE* (1974).

⁹⁰In addition to the above assumptions about the goals of the parties, certain given data can be assumed (which are shown at the bottom of FIGURE 2) in order to provide a more concrete illustration of the model. The given data, however, can be changed to any numbers, and the conclusions of the model will still hold, especially with regard to the basic conclusion that there will generally be convergence when and only when ALD is greater than or equal to ALP.

2. Other Decision and Game Theory Applications

Perhaps the most important value the presentation of this plea bargaining model might have is to stimulate the application of decision theory and equilibrium modeling to legal and political problems other than plea bargaining. Decision theory and equilibrium models are not easy to find in the literature on legal policies and the legal process. That may, however, only reflect the newness of these approaches rather than reflect adversely on their applicability.

Probabilistic decision theory, *i.e.* decision theory under uncertainty, seems to be especially applicable to many of the fundamental decisional problems in the legal process. These include analyzing the behavior of judges in the context of (1) deciding whether to release an arrested person pending trial or confine him to jail by setting a bail figure higher than he can meet, or (2) deciding whether to imprison a convicted defendant or to allow him freedom on a suspended sentence or probation. In the bail context, there are two alternative decisions comprising the rows of four-cell payoff matrix, namely release the defendant or hold him in jail pending trial. The key probabilistic event is the probability of the defendant failing to show up for trial.⁹¹ If the probability is zero that the defendant will fail to show up, meaning that it is virtually certain he will appear in court, then the worst payoff is received if the defendant is held and the best payoff if he is released. If the probability is 1.0 that the defendant will fail to show up, then the next to the best payoff is received if he is held and the next to the worst payoff if he is released.⁹² The order of those payoffs might be changed by one who has less concern for freeing the non-wrongdoer, *i.e.* for

⁹¹Pretrial release statutes specify that the main criterion for release is the probability of appearance. See, *e.g.*, ILL. REV. STAT., ch. 38, § 110-2; FED. R. CRIM. P. 46(c). The model, however, can also consider the probability that the defendant will commit a crime while released, as is discussed in note 93, *infra*.

⁹²For a discussion of how to work with payoff matrices that involve ranking or ordinal data like the above-mentioned pretrial release matrix rather than interval data like our plea bargaining likely sentences matrix, see J. WILLIAMS, *THE COMPLETE STRATEGIST* 198-206 (1954). If as mentioned above in discussing the pretrial release matrix, cell *c* is worst, *a* is best, *d* is good, and *b* is bad, then those cells can be given rank-order payoff scores of $c = 1$, $a = 4$, $d = 3$, and $b = 2$. The two-alternative payoff matrix can be graphed where line one or $LP_1 = a + (b-a)PS = 4 - 2(PS)$, and line two or $LP_2 = c + (d-c)PS = 1 + 2(PS)$. The LP_1 is the likely payoff from holding the defendant; the LP_2 is the likely payoff from releasing the defendant; and PS or just P is the probability of skipping out. Such a strategies graph like FIGURE 1 would involve intersecting pairs of lines. The point of intersection or PS^* is (as in FIGURE 1) the point where $LP_1 = LP_2$ or $4 - 2(PS) = 1 + 2(PS)$. Solving for PS in this equation gives a PS^* of .75. This means that given those cell payoffs, one should release anyone whose PS is below .75 and hold anyone whose PS is above .75. This decision theory analysis would reflect community values even more so if the payoffs were based on a survey, and if the respondents were allowed to indicate relative or interval differences between the cell payoffs rather than just rank orders. Additional data gathering can indicate what percentage of the defendants in a given city fall below PS^* or above PS^* , and what percentage of the defendants are actually released or actually held. The important matter to note, however, is that one does not need to work with absolute units like sentence years or dollars in constructing payoff matrices, but instead can work adequately with relative units or index numbers.

freeing someone who will violate his promise to show up in court.⁹³

Probabilities of skipping out could be obtained for individual defendants or types of defendants.⁹⁴ Payoff values could also be calculated through a survey of relevant persons. The payoff values produced by the survey could be expressed as index numbers or relative distances rather than as absolute units like prison sentences or dollars. The decision theory methodology could be used to determine for any given case or type of case whether the defendant should be released or held in jail.⁹⁵ The decision would depend on whether the expected value of release is greater or less than the expected value of holding the prisoner, by calculating for each case or case type: $a(1-P) + b(P)$ versus $c(1-P) + d(P)$, where P is the probability of skipping out, and a , b , c , and d are the cell payoffs analogous to those in TABLE 1.

If case data is available that closely approximates the data in a given city, an average city, or a type of city, the above decision theory analysis may well result in an extremely low percent of cases in which defendants should be held in light of the probabilities of skipping out and in light of the cell payoffs perceived by the respondents. If so, this might indicate that more arrested persons should be released, or that more arrested persons should be held if the optimum release percentage shown by the decision theory analysis is substantially higher than the actual release percentage. Such a decision theory analysis may also provide insights into the value structures and probability perceptions of arraigning magistrates, regardless whether the decision theory model is close or far from the actual release percentage. A similar analysis could be made of the incarceration/probation decision, except that the key probabilistic event would be the likelihood of the defendant repeating his crime.

Probably the most useful application of the bargaining aspects of decision theory, besides criminal and civil settlement negotiations, is in obtaining a better understanding of certain aspects of contract law. The traditional common law of contract tended to operate on the assumption that (1) the buyer sought to maximize the gap between his upper bargaining price and the settlement price with no conflicting goals, (2) the seller sought to maximize the gap between the settlement price and his lower bargaining price, and (3) both parties had perfect information. The courts refused to interfere with contractual arrangements, arguing implicitly or explicitly that buyers and sellers under such assumptions would both benefit from the equilibrium agreements that would be freely determined.

⁹³As a supplement to the payoff matrix with the probability of skipping out as the key contingent event, one could also have a payoff matrix with the probability of committing a crime (while released) as a second contingent event. This would mean that for a defendant to be released, his probability of skipping out would have to be below PS^* (i.e. the intersecting probability of skipping out discussed in note 92, *supra*) and PW^* (i.e. the intersecting probability of criminal wrongdoing where PW^* is calculated in a manner similar to PS^*).

⁹⁴See Part I, *supra* note 1, Section II-B3(a).

⁹⁵*Id.*

Clarifying those assumptions through geometric and algebraic bargaining theory should help clarify defects in the assumptions and the adverse effects they can produce. Such recognition has led to increased government regulation of contracts dealing with employment, landlord-tenant relations, consumer purchases, and other matters on behalf of the party who is likely to have less information and who is more likely to be influenced by non-economic goals. Even when the assumptions are accurate, the analysis has helped illustrate the need for increased governmental intervention to prevent the parties from contracting to the detriment of the community through such matters as the building of a glue factory in a residential neighborhood or the non-reconstruction of a strip mine. The analysis should also illustrate the method for restoring in a meaningful way the parties to the position where they would have been in a freely negotiated, fully informed agreement which one side has breached.⁹⁶

Closely related to the two-person bargaining in contract law is the multiple-person bargaining associated with coalition formation within collegial courts. This is one area of decision-game theory that has been well developed in the political science literature. Some political scientists have analyzed how the nine judges on the Supreme Court come together in majority and minority coalitions. For example, Glendon Schubert hypothesizes certain goals that each judge or group of judges might have in coalition formation. He then observes their behavior to see if it fits the hypothesized goals. If the behavior does fit the goals, he considers that as providing support for the accuracy of the hypothesized goals, although the behavior could be directed toward other goals or the goals could be achieved by other behavior. If the behavior does not fit the goals, Schubert considers that as a rejection of the hypothesized goals. Although the judges may really have those goals, they may be unable to behave accordingly, because they may misperceive factual means-ends relations or because they may have other conflicting goals.⁹⁷

Walter Murphy, on the other hand, has been among those seeking to explain the dynamic bargaining process which is present on the Supreme Court, although he uses a verbal approach rather than a quantitative one.⁹⁸

⁹⁶See Birmingham, *Damage Measures and Economic Rationality: The Geometry of Contract Law* 1969 DUKE L.J. 49 (1969). See also Birmingham, *Legal and Moral Duty in Game Theory: Common Law Contract and Chinese Analogies*, 18 BUFFALO L. REV. 99 (1969); Birmingham, *A Second Look at the Suez Canal Cases: Excuse for Nonperformance of Contractual Obligations in the Light of Economic Theory*, 20 HASTINGS L.J. 1393 (1969).

⁹⁷See G. SCHUBERT, *QUANTITATIVE ANALYSIS OF JUDICIAL BEHAVIOR* 173-269 (1959); Schubert, *The Study of Judicial Decision-Making as an Aspect of Political Behavior*, 52 AM. POL. SCI. REV. 1007, 1022 (1958); Schubert, *Policy Without Law: An Extension of the Certiorari Game*, 14 STAN. L. REV. 284 (1962).

⁹⁸See MURPHY, *ELEMENTS OF JUDICIAL STRATEGY* (1964); Danelski, *The Influence of the Chief Justice in the Decisional Process of the Supreme Court*, in MURPHY & PRITCHETT, *COURTS, JUDGES, AND POLITICS* 525 (1974); Murphy, *Marshaling the Court: Leadership, Bargaining, and the Judicial Process*, 29 U. CHI. L. REV. 640 (1962).

David Rohde has shown particular concern for testing hypotheses that relate to the size of the winning coalition in the sense that anything more than 5/9 or 56 percent represents possible waste with regard to what judicial principles could have been established, unless an image of unanimity was being sought.⁹⁹ Sidney Ulmer has developed a number of formulas for analyzing the relations between coalition size and the likelihood of error, which has important policy implications for the issue of what minimum size coalition should be allowed for jury convictions.¹⁰⁰

3. Other Equilibrium Modeling Applications

Mathematically, equilibrium models involve two or more simultaneous equations or inequalities that intersect when graphed in such a way that the points of intersection represent the points toward which the behavior of an individual or individuals tends to move. In that sense, probabilistic decision theory is a type of equilibrium model since the intersection or intersections of the highest or lowest payoff lines are points where one's strategy should change from one alternative decision to another. Two-person bargaining and multiple-person coalition models are thus also equilibrium models since they often hypothesize a point toward which the bargainers gravitate or a point that represents a kind of natural coalition like the minimum winning coalition. The most common equilibrium models in elementary economics are the supply and demand model, the consumer model, and the firm model. All three have numerous potential applications to legal policy problems.¹⁰¹

The supply and demand or market model says that there are two simultaneous equations which determine the equilibrium price and equilibrium quantity in a competitive market. One equation shows quantity demanded as a function of price, and the other equation shows quantity supplied as a function of price. Where those two equations intersect is where the prevailing price and quantity bought and sold will tend to be. This is so because at any higher price suppliers will supply a larger

⁹⁹See Rohde, *Policy Goals and Opinion Coalitions in the Supreme Court*, 16 MIDWEST J. POL. SCI. 208 (1972); Rohde, *A Theory of the Formation of Opinion Coalitions in the U. S. Supreme Court*, in PROBABILITY MODELS OF COLLECTIVE DECISION-MAKING (R. Niemi & H. Weisberg eds. 1972). Rohde draws heavily upon W. RIKER & P. ORDESHOOK, POSITIVE POLITICAL THEORY (1973) and W. RIKER, THE THEORY OF POLITICAL COALITIONS (1962).

¹⁰⁰See ULMER, COURTS AS SMALL AND NOT SO SMALL GROUPS (1971); Ulmer, *Quantitative Analysis of Judicial Processes: Some Practical and Theoretical Applications*, 28 LAW & CONTEMP. PROB. 164, 176 (1963); Zeisel, . . . *And Then There Were None: The Diminution of the Federal Jury*, 38 U. CHI. L. REV. 710 (1971). Political scientists have also studied such matters as the measurement of cohesion in judicial coalitions, the extent to which judges with similar background characteristics tend to be in the same coalitions, and the measurement of the pivotal power or other power indices of coalitions. Much of this literature is reviewed in the first Ulmer article above. Most of it, however, does not contain a bargaining element or a deductive mathematical model like that associated with decision-game theory.

quantity than buyers will buy, and suppliers will thus lower their selling price to sell the surplus. At any lower price buyers will demand a larger quantity than suppliers will supply, and buyers will thus raise their buying price to encourage more production. This model can be used to get a better partial understanding of how the salaries of legal process personnel like judges, prosecutors, and public defenders are determined. Those salaries may in turn have a substantial influence on the quality of legal process personnel. This model can also be used to get a better partial understanding of the market price for prostitution, gambling activities, stolen goods, hired killers, and other criminal services. Indeed, a whole school of criminology may be in the process of developing which emphasizes that the way to decrease crime is to increase the costs of committing crimes, including the missed opportunity costs (*i.e.* shift the supply curve to the left so that at a given price, less will be supplied because of higher costs), and decrease the benefits which cause people to demand or seek criminal activities, *i.e.* shift the demand curve to the left so that a given price, less will be demanded because of lower benefits from the product.¹⁰²

The consumer model says that the consumer is in optimal equilibrium when the marginal benefit-cost ratio of every product that he buys is equal to the marginal benefit-cost ratio of every other product that he buys. The marginal ratio of a product or good is the ratio of the benefit to be received from buying one additional unit to the cost to be paid for that one additional unit. If the marginal benefit-cost ratios are not all equal, then the consumer should buy more of those products which have higher marginal or incremental ratios and less of those products which have lower ratios. This model applies to many legal policy problems where one is seeking to get an optimum mix among a variety of goods.¹⁰³ All of these optimum mix situations involve somewhat difficult problems in measuring benefits or satisfaction and measuring costs or effort. The measurement methods however, may be less difficult than those involved in measuring

¹⁰¹For materials which deal with multiple applications of economic modeling to law, see G. BECKER & W. LANDES, *ESSAYS IN THE ECONOMICS OF CRIME AND PUNISHMENT* (1974); G. TULLOCK, *THE LOGIC OF THE LAW* (1971). For those who think they need an introduction to or review of basic economic models of the market, the consumer, or the firm, see any elementary economics textbook such as P. SAMUELSON, *ECONOMICS* (1972).

¹⁰²See R. ANDREAS & J. SIEGFRIED, *THE ECONOMICS OF CRIME* (forthcoming); L. KAPLAN, *AN ECONOMIC ANALYSIS OF CRIME* (1976); A. ROGERS, *THE ECONOMICS OF CRIME* (1974); G. TULLOCK, *THE ECONOMICS OF CRIME* (forthcoming); *THE ECONOMICS OF CRIME AND PUNISHMENT* (S. Rottenberg ed. 1973); Becker, *Crime and Punishment: An Economic Approach*, 76 J. OF. POL. ECON. 169 (1968).

¹⁰³Examples include finding an optimum mix between (1) law reform and case handling in the OEO Legal Services Program, (2) free press and fair trial in prejudicial pretrial press reporting, and (3) voting, schools, criminal justice, housing, public accommodations, and employment civil rights activities in promoting equality improvement. See S. NAGEL, *MINIMIZING COSTS AND MAXIMIZING BENEFITS IN PROVIDING LEGAL SERVICES TO THE POOR* (1973); S. NAGEL & M. NEEF, *THE APPLICATION OF MIXED STRATEGIES* (1976); Nagel, Reinbolt, & Eimermann, *A Linear Programming Approach to Problems of Conflicting Legal Values Like Free Press Versus Fair Trial*, 4 RUTGERS J. OF COMPUTERS AND THE LAW 420 (1975).

the plea bargaining cell payoffs, the conviction probabilities, and especially the non-sentence goals.

The model of the firm says that a one-product firm is in optimal equilibrium such that its total profits will be maximized when its marginal revenue equals its marginal cost. Marginal revenue or benefit is the additional unit of income which comes from producing one additional unit of the firm's product. Marginal cost is the additional unit of cost which comes from producing one additional unit of the firm's product. If marginal or incremental revenue were greater than marginal cost, then the firm should keep producing more units of its product, because more income than cost has been produced from each additional unit. If marginal revenue were less than marginal cost, then the firm should cut back on the number of units it produces because at least the last few units have been costing more to produce than the revenue they generate. This equilibrium model may have more applicability to legal policy problems than any of the previous equilibrium models. It could help provide a better understanding of what is involved in finding the optimum level of due process to produce in criminal and civil cases. The optimum level would be the point where the marginal cost of convicting the innocent, which can be considered a negative benefit or negative marginal revenue, equals the marginal cost of acquitting the guilty. This same firm model could also help provide a better understanding of what is involved in finding the optimum level of severity for economic regulation laws, divorce laws, and other substantive laws. The optimum level would be the point where the marginal cost of being under-severe (which is like negative marginal revenue) equals the marginal cost of being over-severe.¹⁰⁴

As with the plea bargaining model, one may have to resort to non-precision forms of measurement in order to apply the model of the firm, the consumer, and the market to legal policy problems. Even then, however, these models can be insight-provoking. They provide insights for comparing optimum or equilibrium behavior with empirical behavior so that one can make policy recommendations to bring the empirical closer to the optimum, or so that one can revise the values he attributes to the policy-makers in order to bring the optimum closer to the empirical. They provide insights for understanding the effects on other variables of changing legal policies and decisions, and the effects on legal policies and decisions of changing other variables. They help to clarify assumptions, goals, alternative means, payoffs from alternative means, contingent probabilities, and other elements essential to understanding more fully the basic simplicities and subtle complexities of law and the legal process.

¹⁰⁴S. NAGEL, P. WICE, & M. NEEF, *THE POLICY PROBLEM OF DOING TOO MUCH OR TOO LITTLE* (1976); R. RIDKER, *ECONOMIC COSTS OF AIR POLLUTION* (1967); Posner, *supra*, note 52.

APPENDIX 1: GLOSSARY OF SYMBOLS

SYMBOL	REPRESENTS	SECTION FIRST APPEARING
<i>Parties:</i>		
D	The defendant or defense counsel	II-A1
P	The prosecutor	II-A1
<i>Defendant's Alternatives:</i>		
Alt. #1 or T	Defendant goes to trial	II-A1
Alt. #2 or J	Defendant pleads guilty before a judge without a bargain	II-A1
Alt. #3	Defendant settles with the prosecutor	II-A1
F	Coin flip or other random method to determine whether defendant goes to trial or pleads before a judge	III-A2(b)
MR	Method of non-bargain resolution (trial or non-negotiated plea)	II-A2
<i>Payoff Cells</i>		
<i>a</i>	Cell showing perceived sentence if defendant pleads guilty when probability of conviction is low	II-A1
<i>b</i>	Cell showing perceived sentence if defendant pleads guilty when probability of conviction is high	II-A1
<i>c</i>	Cell showing perceived sentence if defendant goes to trial and is acquitted	II-A1
<i>d</i>	Cell showing perceived sentence if defendant goes to trial and is convicted	II-A1
<i>d'</i>	True value of cell <i>d</i> known only to an omniscient being	IV-B3 (note 77)
<i>d' '</i>	Sentence which is deserved by the defendant in light of his characteristics and in light of the crime	IV-B3 (note 77)
<i>Utility Units:</i>		
DIS	Dissatisfaction units or negative satisfaction units in a payoff cell, not just sentence years	II-A3
SAT	Satisfaction units in a payoff cell, not just sentence years	II-A3

Probabilities:

PC	Perceived probability of defendant being convicted	II-A1
PC'	True probability of conviction as known by an omniscient being	II-B3(a)
PCD	Probability of conviction as perceived by the defendant	II-B3(a)
PCP	Probability of conviction as perceived by the prosecutor	II-B3(a)
PG	Probability of whether or not the defendant is actually guilty	II-B3(a)
PL	Perceived probability of liability being established in a civil suit	IV-A1
PS	Probability of defendant skipping if released	IV-C2 (note 92)

Threshold Probabilities:

PC*	The probability of conviction on trial that makes going to trial or pleading guilty without a bargain seem like equally desirable alternatives	II-B2(c)
PS*	The probability of a released defendant showing in court that makes holding or releasing him prior to trial seem like equally attractive alternatives	IV-C2 (note 92)
PW*	The probability of a released defendant committing a wrong or a crime that makes holding or releasing him prior to trial seem like equally attractive alternatives	IV-C2 (note 93)

Sentences and Settlements:

LS	Likely sentence in years or expected sentence value from either trial or a non-negotiated plea	II-A1
LS ₁	Likely sentence from going to trial	II-B1
LS ₂	Likely sentence from a non-negotiated plea	II-B1
LS*	Likely sentence at the point where going to trial or pleading without a negotiated bargain are equally attractive alternatives	II-B2(c)
R	Result of clash between defendant and prosecutor	III-A2(b)

S*	Likely settlement at the point of equilibrium or the convergence point between the adjusted limits of the defendant and prosecutor, where defendant's limit is higher than the prosecutor's	III-A1(b)
S'	The true settlement as known to an omniscient being	IV-B1

*Bonus Factor,
Defendant:*

XD	Defendant's bonus factor in bargaining to be added to the defendant's unadjusted bargaining limit to obtain ALD	II-B4(b)
%XD	Defendant's percentage bonus factor to be multiplied by the defendant's unadjusted bargaining limit to obtain XD	II-B4(b)

*Discount Factors,
Prosecutor:*

XP	Prosecutor's discount factor in bargaining, to be subtracted from the prosecutor's unadjusted bargaining limit to obtain ALP	II-B4(b)
%XP	Prosecutor's percentage discount factor to be multiplied by a prosecutor's unadjusted bargaining limit to obtain XP	II-B4(b)

Bargaining Limits:

ALD	Adjusted bargaining limit of the defendant	II-B4(b)
ALP	Adjusted bargaining limit of the prosecutor	II-B4(b)
L	Bargaining limit for the defendant or the prosecutor	III-B3(b)
LD	Defendant's unadjusted bargaining limit	II-B1
LP	Prosecutor's unadjusted bargaining limit	II-B1

Attitudes of Bargainers:

M	Middling attitude on the optimism-pessimism scale	III-A2(a)
O	Optimistic attitude	III-A2(a)
O-P coefficient	Optimism-Pessimism coefficient on a 0 to 1.00 scale	II-B3(c)
P	Pessimistic attitude	III-A2(a)

Exaggeration Factors

EFD	The fraction less than one by which the defendant multiplies his adjusted limit in order to arrive at an initial offer	III-B1
EFB	The number more than one by which the prosecutor multiplies his adjusted limit in order to arrive at an initial offer	III-B1

Offer (Symbol can be preceded by a D or a P):

O_{i0}	Initial offer, or offer at time 0	III-B1
O_{i1}	First counter offer	III-B1
$O_{i2} \dots O_{in}$	Subsequent bargaining offers	III-B1
O_{ii}	Counter offer at any time i or at any stage	III-B3(b)
O_{ii-1}	Counter offer of any previous stage, or time i minus one time unit	III-B3(b)

Splitting Rates:

R	Splitting rate for the defendant or the prosecutor	III-B3(b)
RD	Splitting rate in defendant's bargaining, or the fraction of the distance between his last offer and his bargaining limit	III-B1
RP	Splitting rate in prosecutor's bargaining, or the fraction of the distance between his last offer and his bargaining limit	III-B1

Time Designation:

t	Time (used as a subscript like i and n)	III-B1
i	Time point i or stage i corresponding to any stage	III-B3(c)
n	The last time point	III-B3(c) (note 44)
T	Stage number from 1 to N (used as a variable)	III-B3(c) (note 44)

Benefits and Costs:

B	Benefits	IV-B3 (note 74)
B/C ratio	Benefit/Cost ratio	IV-C3
C	Costs	IV-B3 (note 74)
\$L	Litigation costs in a civil suit	IV-A1
\$S	Settlement costs in a civil suit	IV-A1

*Discounting Future**Value of Settlements:*

A	Amount or future value of a future payoff	IV-A2
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P	Principal or present value of a future payoff	IV-A2
r	Interest rate of an investment minus the inflation rate	IV-A2

Variables, Types of:

X	Independent variable used to predict from	II-B1
Y	Dependent variable to be predicted to	II-B1

Regression Coefficients:

A or a	Value of the variable being predicted to when the variables being predicted from have zero values in a linear or log-linear relation	II-A2
B or b	Ratio between a change in the variable being predicted to one a one-unit change in the variable being predicted from, holding other variables constant	II-A2

Miscellaneous:

CS	Measure of crime severity based on sentences provided in the statutes	II-A2
P ₁	Proportion of all cases which go to trial	Footnote 34
P ₂	Proportion of all cases which plead guilty before a judge without a bargain	Footnote 34

APPENDIX 2: BASIC FORMULAS USED

- (1) Expected values (or likely sentences associated with each choice)

$$\begin{aligned} LS_2 &= (1-PC) (a) + (PC) (b) & LS_2 &= a + (b-a)PC \\ LS_1 &= (1-PC) (c) + (PC) (d) & LS_1 &= c + (d-c)PC \end{aligned}$$
- (2) Non-adjusted bargaining limits

$$LD \text{ or } LP = LS_1 \text{ or } LS_2, \text{ whichever is lower}$$
- (3) Adjusted bargaining limits

$$\begin{aligned} ALD &= LD + XD & XD &= (\%XD \cdot LD) \\ ALP &= LP - XP & XP &= (\%XP \cdot LP) \end{aligned}$$
- (4) Likely settlement

$$S^* = .5 (ALD + ALP)$$
- (5) Threshold PC and LS (where either going to trial or pleading guilty without a bargain is equally attractive)

$$\begin{aligned} PC^* &= (a-c)/(a-b-c+d) \\ LS^* &= (ad-bc)/(a-b-c+d) \end{aligned}$$
- (6) Offers
 - (a) Initial offer

$$DO_{t0} = EFD \cdot ALD \qquad PO_{t0} = EFP \cdot ALP$$
 - (b) First counter-offer

$$O_{t1} = O_{t0} + RD(ALD - O_{t0}) \qquad O_{t1} = O_{t0} - RP(O_{t0} - ALP)$$
 - (c) General counter-offer in terms of prior counter offer

$$O_{ti} = O_{ti-1} + R(L - O_{ti-1})$$
 - (d) General counter-offer in terms of initial offer

$$O_{ti} = L + (1-r)^{ti} (O_{t0} - L)$$
- (7) Present value in time discounting

$$P = A/(1+R)^t$$

