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Notes on a Grand Illusion: Some Limits on the Use of Bayesian Theory in Evidence Law

By CRAIG R. CALLEN*

It is now apparent that the concept of a universally accepted, infallible body of reasoning—the mathematics of 1800 and the pride of man—is a grand illusion. Uncertainty and doubts concerning the future of mathematics have replaced the certainties and complacency of the past. . . . The present state of mathematics is a mockery of the hitherto deep-rooted and widely reputed truth and logical perfection of mathematics.1

If criticisms of the rules of evidence or of the results of controversial trials have any point, there must be a set of principles against which rules or trials may be appraised. Unless there are reasonably ascertainable standards against which one can assess the litigation process, criticism of the law of evidence or the outcome of trials is only disagreement over matters of personal preference.2

Scholars in the last two decades, marching under the banner of Bayes' Theorem,3 have argued that the rules of evidence4 and the decisionmak-
ing of factfinders\textsuperscript{5} can and should conform\textsuperscript{6} to mathematical probability theory. They argue that a system of factfinding which does not rely on an articulable standard of accuracy is a system which necessarily treats similarly situated persons differently without justification.\textsuperscript{7}

This article will show that the application of Bayes' Theorem as a mathematical model for evidence rules and for the trial process entails

\textsuperscript{5} Kaye, \textit{Res Ipsa}, supra note 3, at 1457.


\textsuperscript{7} Professors Ellman and Kaye appear to concede that deviation from a Bayesian model in factfinding is proper. Ellman & Kaye, supra note 5, at 1153-54. The principle according to which these deviations may be made is, however, hard to discern.
problems not addressed in the legal literature, along with a number of problems which have been addressed elsewhere. Particular defects in Bayesian theory include its doubtful psychological assumptions, the problems it poses for the relationship between philosophy of knowledge (or epistemology) and the law, and the distributive justice problems which certain applications of the theorem will create.

The argument is not that Bayesian theory is useless as a means of analyzing the factfinding process, but rather that, contrary to the impression created by its advocates, Bayesian theory is inadequate as a model for rules of evidence. This article suggests as a standard of equal, if not greater, importance in assessing evidence rules, a concept here called "verification value" which is derivable from a number of rules of evidence, including the hearsay and character rules. Analysis of the trial process or rules of evidence in light of the verification value of the information involved illuminates the relationship among a number of rules of evidence and alleviates the problems which exclusive reference to Bayes' Theorem creates.

AN INTRODUCTION TO BAYES' THEOREM

Bayesian theory, also known as subjective probability, is a rigorous, internally consistent system based on the idea that one can assign a numerical value to one's inferences and combine those values to achieve the same kind of results as one would with "objective" probabilities. Subjective probability theory differs from objective probability theory in that the latter assigns values based on the results of numerous observations of past events, while subjective probability requires no such observations.

One distinction is important here. The factfinding process, including the rules of evidence to the extent they are rules of investigation for the factfinding process, is an inductive, not a deductive, process. An example of deductive reasoning is:

All killings of wives are committed by their husbands.
The deceased was a wife and was killed.
The accused was her husband. Therefore, the accused killed the deceased.15

An inductive analog is:

In most cases of which society is aware, where wives were killed, their husbands were the killers.
The deceased was killed and a wife.
The accused was her husband.
No reason appears to distinguish this case from most cases.
Therefore, the accused killed the deceased.16

In deductive reasoning, if the premises (the propositions preceding the conclusion) are true, the conclusion must be true. In inductive reasoning, even if the premises are true, the conclusion may be false.17 Induction is the process of making inferences based on evidence which is inconclusive.18 In order to understand why subjective probability is not ideal for induction, that is to say, why subjective probability is not a system of inductive reasoning which provides the correct result in all cases,19 one first must consider how Bayesian theory works.

Selecting a Probability Value

The first step in applying Baye’s Theorem to factfinding is assignment of a numerical value to one’s degree of belief in an inference which is based on specific evidence and which states whether a particular contention is true or false.20 Unless this assignment process is sound, the mathematical model for the jury’s deliberations has no particular usefulness. Unless one can say with some confidence, that an inductive inference from a piece of evidence yields a probability, for example, of .7 that a particular conclusion is true or false, calculations based on that numerical value are likely to be pointless.21

16 See id.
17 One could avoid this problem by identifying relevant distinctions and stating that none existed in this case, and including the premise that conditions would not change over time. That would convert the reasoning into deductive reasoning.
18 Popper holds that inductive inferences cannot be justified because an attempt to justify induction depends on induction—the very thing to be justified. K. POPPER, supra note 13, at 28-30. This article nevertheless uses induction as the process of making inferences from inconclusive evidence because this appears to be common, if not universally accepted, philosophical and logical usage, see, e.g., G. HARMAN, supra note 15, at 164-68.
19 While no one seems prepared to argue this proposition, it must be initially explained to enable explanation of the remainder of the article. See notes 30-42 & accompanying text infra.
20 As to whether “true” or “false” should be considered the only possible choices, see note 65 infra. Readers who want to look at a use of the theorem should examine notes 43-47 & accompanying text infra.
21 An analogy would be to a syllogism valid under the conditions of formal logic, but containing false premises:
Subjective certainty is neither precisely prescribed by the individual nor entirely under the control of that individual. One might be subjectively certain that God exists, for example, and yet be unable to decide what corroborative weight various data have in establishing that conclusion. Selecting a numerical equivalent for the likelihood of a given result from certain evidence requires that subjective certainty, or the maximum possibility of belief, be considered to be at one end of a scale—e.g., that subjective certainty be taken as equivalent to 1.0, that total disbelief be taken as equivalent to 0, and that the degree to which one believes an assertion to be true is measured on the scale between 0 and 1.0. Whether a reliable gradation can be anticipated is problematic.

Subjective probability theorists have attempted to solve this gradation problem by positing that the factfinder can select the proper probability by analogy to betting or odds-making. This practice is not a real improvement over unaided selection of gradations. Many people, although they may have some notion of the chances of drawing one given card from a pack of 100, have no experience precisely fixing the odds of betting on more complicated propositions. Requiring factfinders to engage in a type of activity in which they never before have engaged, or in which they have engaged only rarely, on the theory that this will "improve" reasoning in which they have regularly engaged and have had some opportunity to test, is dubious. Moreover, a subjective probability figure, on a given set of data, is only the factfinder's estimate of the mathematical probability that a given result will occur. That the factfinder makes an estimate does not of itself make the estimate useful.

Suppose the judge instructs a jury, of which J is a member, according to Bayesian theory. In order for J and the other jurors accurately to assess

The sky is green.
All things which are green are on Mars.
\[\therefore\] The sky is on Mars.


B. RUSSELL, HUMAN KNOWLEDGE 396-98 (1948).

There are several problems with this assumption. First, one is likely to change the odds one would be willing to fix given what one is risking. L. COHEN, THE PROBABLE AND THE PROVABLE 91 (1977). Second, to the extent one postulates what is at risk, one might not assign a risk adequately corresponding to the adverse effect an incorrect verdict would have on the parties. See Kaplan, supra note 3, at 1067-69. Third, to the extent one establishes hypothetical bets or other simulation devices as part of the jury's decisionmaking, one runs the risk of having the jury take its duty less seriously than is desirable. Cf. Tribe, supra note 2, at 1375-77 (use of mathematical model for jury decisionmaking may discourage jury from employing intuition and common values it was designed to provide by making jury process too mechanical). The most telling exception, the one on empirical grounds, is discussed in the text accompanying notes 24-29 infra.

Cf. I. LEVI, GAMBLING WITH TRUTH 47-49 (1967).

Cf. K. POPPER, supra note 13, app., at 387-91 (arguing that degree of corroboration cannot be identified with probability, a distinction discussed more fully at notes 100-14 & accompanying text infra).
subjective probabilities of whatever class of behaviors is considered the class comparable to the behavior at issue, J and the other jurors would have to participate in a great deal of empirical testing. For example, if J selects .75 as the subjective probability of X, and if the "correct result" in this test case is not-X—that is, that X is not true—J's selection of .75 is not necessarily an incorrect subjective probability estimate. Instead, the particular case might be one of the cases included in J's estimate of a .25 probability of not-X. In order to have reasonable confidence as to the reliability of J's estimate, his accuracy would have to be tested over a fairly large number of cases.

This poses both practical and theoretical problems. The major practical problem is the increase in time and expense of litigation which results from such testing or experience. The major theoretical problem is that of requiring the court to set standards for the tests, and to govern the jurors' adjustment to the results of those tests. Courts are ill-adapted to govern this adjustment. J's nonmathematical inductive ability has been tested through the duration of his life. By the time of the trial, he has some idea of the possibility for error contained in one of his inductions. Given the unlikelihood that J can accurately adapt to the use of mathematical subjective probabilities, the use of unaided induction seems preferable.

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26 The number and sample structure (given the variations of their positions which the parties may assert at trial) of such replications would probably be a matter which a trial judge considered himself unable to resolve. Callen & Kadue, To Bury Mutualty, Not to Praise It: An Analysis of Collateral Estoppel After Parklane Hosiery Co. v. Shore, 31 Hastings L.J. 755, 769 (1980).

27 Such testing may even be impossible. K. Popper, supra note 13, at 191; Brilmayer & Kornhauser, supra note 3, at 136.

28 The exact formula and means of correction is a question with which trial judges may be similarly poorly adapted to deal. Cf. Callen & Kadue, supra note 26, at 769 (discussing impossibility of judicial control for influence of extrajudicial factors on a number of trials).

29 See generally Grant, Knowledge, Luck and Charity, 89 Mind 161 (1980). Professor Kaye makes the somewhat unusual argument that sodium pentothal, polygraphs, and torture are available after the outcome of a trial to determine whether the jurors' conclusions are incorrect, so jurors' use of subjective probability theory is not, in principle, unverifiable. Kaye, Laws, supra note 3, at 45 n.41, 53 n.60. This is inappropriate. Insofar as one means to include particular jurors in a Bayesian trial, he must have confidence in their use of Bayesian theory, or at least their ability to play roles the theory requires of them. This confidence would only be derived by prior experience with them, or with an extremely broad representative group. Professor Kaye's verifiability suggestions, besides being unconscionable, are methodologically crude in that if one acts according to his suggestion, one may find out whether a criminal is guilty but not know whether the jury used subjective probability accurately in reaching its verdict, or whether and to what extent the result deviated from the result of "intuitive" methods.

Some commentators hold that a paucity of particularized evidence supporting a party's contention that C is true of his opponent should result in a reduction in the subjective probability that C is true as compared with the probability based on more general evidence. In other words, these commentators argue that if there is evidence that would lead one to believe that the probability that C is true of all persons in class S is .51 and if the
The doubtful worth of fixing values according to Bayesian precepts is perhaps the least acute problem with the use of subjective probability to govern induction by factfinders. An example which shows at least one more acute problem is a hypothetical litigation which will recur in this article and is referred to here as the blasting cap case.

The Blasting Cap Case

Suppose that the question is whether B was injured by a defective blasting cap manufactured by company $M_1$ or manufactured by another manufacturer. All parties concede that the blasting cap was defective. The evidence shows that the blasting cap was three-pronged and was produced by the turbo-system process. The evidence further shows that $M_1$ manufactures ninety-five percent of all three-pronged blasting caps manufactured by the turbo-steam process and the other five percent are manufactured by $M_2$, $M_3$, and $M_4$. Distribution is uniform throughout the world. Given the explosion of the blasting cap, no further evidence as to the identity of the manufacturer can be obtained.

party only goes so far as to prove that his opponent is in class $S$, and offers no explanation for his lack of more particularized proof that $C$ is true of his opponent, the subjective probability that $C$ is true of the party's particular opponent is lower than .51. The rationale for this argument is that the lack of particularized evidence is itself important in deciding whether $C$ is true of the opponent. Kaye, The Paradox of the Gatecrasher and Other Stories, 1979 ARIZ. ST. L.J. 101, 107-08 [hereinafter cited as Kaye, Gatecrasher]; Tribe, supra note 2, at 1346 n.55, 1349-50. This argument is explored more fully in notes 71-87 & 171-82 & accompanying text infra. Based on this argument, one could infer that lack of experience in estimating subjective probabilities should likewise result in an adjustment in the probabilities to account for uncertainty. Bertrand Russell has said of this prospect:

[Certain comparisons suggest] that the uncertainty of data is quantitative, and can be equal or unequal to the uncertainty derived from a probability inference. . . . In practice the numerical measurement of the uncertainty of a datum is seldom possible. We may say that uncertainty is a half when the doubt is such as to leave an even balance between belief and disbelief. But such a balance can only be established by introspection, and is incapable of being confirmed by any sort of test.

B. RUSSELL, supra note 22, at 395. Such an adjustment also presents problems in terms of the requirement of additivity. See notes 62-66 & accompanying text infra.

Assume that the only question in $B$ v. $M$ would be whether $M_1$ manufactured the blasting cap which injured $B$. This hypothetical is very similar to the blue bus case which Professor Tribe poses. Tribe, supra note 2, at 1340-41, 1346-50; see text accompanying note 149 infra. This article varies from Tribe's hypothetical because Tribe's discussion of his hypothetical is directed primarily toward distributive justice and sufficiency of evidence considerations. The facts were varied to avoid implication of those questions into this part of the article, which is basically an epistemological discussion. See generally Brilmayer & Kornhauser, supra note 3, at 120, 130-31. The issues of sufficiency of evidence, distributive justice, and admissibility which the blasting cap case raises are considered in notes 69-87 & accompanying text infra.

The facts in this problem are reminiscent of Hall v. E.I. du Pont de Nemours & Co., 345 F. Supp. 353 (E.D.N.Y. 1972), which "suggested" application of an industry-wide liability theory, on somewhat stronger facts, against blasting cap manufacturers. Sindell v. Ab-
If induction by factfinders at trial is a process governed by subjective probability, then in every like case, the jury would be required to find that $M_1$ manufactured the blasting cap.\footnote{This assumes that like cases will occur—a plausible assumption when the product at issue is blasting caps. The probability here is subjective in that one is required to estimate, \textit{inter alia}, the probability that $M_1$ manufactured the blasting cap at issue based on the percentage of all blasting caps which $M_1$ manufactured rather than the percentage of all defective blasting caps which $M_1$ manufactured. See note 33 & accompanying text \textit{infra}.} Given the destruction of the blasting cap, no inference arises, from $B$'s inability to produce other evidence, that $B$ is concealing adverse evidence concealment of which might make $B$'s case an exception to the requirement of an inference of manufacture by $M_1$ of the cap in issue from $M_1$'s total proportion of manufacture of blasting caps.\footnote{For authorities relying on failure to produce additional evidence as a basis for an inference adverse to the proponent's case, see, for example, Kaye, \textit{Gatecrasher}, supra note 29, at 107, 108; Tribe, \textit{supra} note 2, at 1349-50. Tribe would probably argue for a directed verdict in this situation. Tribe, \textit{supra} note 2, at 1361 & n.102. While a directed verdict may be called for here, it is basically a question of the principle on which liability is to be founded rather than a question of the application of a Bayesian system. See notes 69-87 & accompanying text \textit{infra}.} $B$'s lack of additional proof would not entail a lower subjective probability than ninety-five percent and that probability would require an inference of manufacture by $M_1$ in this and all similar cases. Yet, $M_1$ did not manufacture all such caps.\footnote{See notes 30-31 & accompanying text \textit{infra}.} Subjective probability will not yield a correct result in every case. Therefore, subjective probability is not a system which invariably provides the only correct result.

Two points follow from this example. First, given that subjective probability does not necessarily include checks against an incorrect initial probability, accuracy in setting the numerical value of a subjective probability is important.\footnote{This point is important to keep in mind in distinguishing the use of what Professor Kaye calls "classical" statistics, Kaye, \textit{Laws}, supra note 3, at 51 n.57, from Bayesian theory in litigation. When an action is claimed to have been taken with regard to a \textit{class}, application of "classical" statistics to determine whether that class shows the effect of such an action mitigates problems of inaccuracy that individualized use of statistics may create.} Second, subjective probability must derive whatever value it has as a test of the trial process from application across a broad class of cases.\footnote{One might ask whether the issue is the percentage of all \textit{similarly defective} blasting caps manufactured by $M_1$ rather than the percentage of all blasting caps manufactured by $M_1$. See generally Tribe, \textit{supra} note 2, at 1365-66. To ask about all caps discourages $M_1$'s competitors from special quality control measures, as their liability for defects is minimized by their low market shares (which, ironically, might be due in whole or part to $M_1$'s quality control). See notes 20-29 & accompanying text \textit{infra}; notes 79-81 & accompanying text \textit{infra}.} However, application of subjective probability in a class of cases...
involves certainty of error in a number of those cases, namely, those in the statistical minority, and the severe risk of other errors based on defects in the Bayesian scheme such as fixing of probability estimates.

While subjective probability may not yield the one and only accurate result, it does not follow that application of subjective probability theory would not yield the accurate result in more cases than mere adherence to the rules currently governing factfinding. One may believe, of course, that those who are generally accurate in their inductions are really using subjective probability theory, even with its inherent limitations. Five considerations make this implausible.\footnote{First, coherent non-Bayesian theories of induction have been developed,\footnote{Compare Griggs v. Duke Power Co., 401 U.S. 424 (1971) (holding that under Title VII of the Civil Rights Act of 1964, 42 U.S.C. §§ 2000e to 2000e-16 (Supp. III 1979), employment test which has not been shown to be job-related, although neutral on its face, and which operates to bar blacks from employment disproportionately as shown by objective statistics, is actionable racial discrimination), with McDonnell-Douglas Corp. v. Green, 411 U.S. 792, 805-07 (1973) (Court reasoned that if black applicant is rejected for reemployment because of his involvement in illegal stall-in to protest alleged racial discrimination and that reason for rejection is not pretext, even though such refusals to rehire would have disproportionate impact on blacks, such refusal would not be actionable under Title VII; Court implicitly refused to rely solely on statistics when individual's conduct was basic issue).} establishing that non-Bayesian induction is conceivable. Thus, it does not necessarily follow that those who are generally accurate in their inductions are really using subjective probability theory. Second, some of the leading “inducers” argue that non-

\footnote{The question whether Bayesian calculations are more accurate than ordinary “intuitive” calculations must be answered in terms of plausibility because empirical testing is impossible. Brilmayer & Kornhauser, supra note 3, at 136. Although Professor Kaye has attempted to refute this point, the test he proposes would be both unconscionable and methodologically flawed. See note 29 supra.}
mathematical methods of intuition are the key to their inductions. Third, in the blasting cap example, the subjective probability is reached without a consideration of the percentage of defective caps produced by \( M_1 \); the subjective probability of ninety-five percent is based, perhaps erroneously on \( M_1 \)'s share of the market; there is no distinction between the percentage of defective and nondefective caps produced. Fourth, use of subjective probability entails errors in all cases which are in the statistical minority. Fifth, if the unconscious Bayesian theory were true, then it seems likely that reliable inducers would recognize and reproduce the subjective probability theory mathematics underlying their inductions with little difficulty. A number of those inducers do not believe that their inductive conclusions are mathematically based. It is not particularly uncommon to find that a person who makes sound generalizations is unable to engage in Bayesian calculations or calculations of comparable complexity. Therefore, a person may be able to make sound inductive conclusions without unconsciously using Bayesian theory. It follows that the "unconscious mathematics" theorists either are in error or have discovered a class of idiots savants, including Einstein and Edison, much larger than any previously supposed.

The Problem of Dependency

Complex cases would involve difficult testing to determine whether there is at least apparent conformity to mathematical theory. A hypothetical illustrates the problem. Suppose two political assassinations occur simultaneously at point X in a crowded metropolitan area at 1 p.m. News of the assassinations is broadcast immediately. Assume the crimes were committed by two different criminals, L and M. All witnesses concur that L killed victim V, and ran to a car of a given make, model, year, and color, got in, started the car, and was immediately pursued by police in car 45. The police in car 45 testify that they pursued the car (their description of the car matches that of the witnesses and extends to decals in the rear window, the license plate number, and a stuffed animal on the ledge above the back seat of the car) and that it was in fact L's car. The officers testify that from the rear the driver of the auto resembled L. The officers further state that before the driver of the car eluded them, they chased it going due west from point X for fifteen minutes at speeds

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38 For example, Albert Einstein, quoted in K. POPPER, supra note 13, at 32, and Thomas Edison, quoted in R. CONOT, A STREAK OF LUCK 142 (1979).
39 See note 33 supra.
40 See note 35 & accompanying text supra; Tribe, supra note 2, at 1349-50.
41 Note 40 supra; Kaye, Laws, supra note 3, at 45.
42 See note 38 supra.
exceeding seventy miles per hour. Suppose a jury was asked to assign a subjective probability of L's guilt based on this testimony.

M walked into his office at 1:20 p.m. Before leaving, he told his secretary he was going out to see some clients. It is conceded by all parties that when he arrived in his office, M stated "I killed victim W, he deserved it, and I'm glad I did it." Suppose a jury on the second case was asked to assign a subjective probability of M's guilt based on this evidence.

The problem of dependency may be illustrated as follows: Assume now that only one homicide was committed, that only one criminal was involved, that only one defendant is being prosecuted, that he is the defendant who supposedly confessed, and that the auto referred to is identified by the police as the lone defendant's auto. The testimony about the criminal's flight and the car chase and the testimony about admission could both be true. If both are true, in order for the (now one and only) defendant to be guilty, the defendant would have to have driven from the point where the car chase terminated, at least 17.5 miles west of point X, to his office in five minutes. Suppose the office is five miles east of point X. This would mean defendant had driven approximately 270 miles per hour through a congested metropolitan area for 22.5 miles between 1:15 and 1:20 p.m. This would be a highly unlikely occurrence, to say the least. Thus, if one of the stories is true, the other is quite likely to be false. If both stories are true, it is likely that someone who resembled the defendant was driving the defendant's car, and that the defendant is not guilty. Inferences as to guilt and to the truth of the two stories are dependent on one another.

In this case, subjective probability theory would require the factfinder at least in effect to determine the following subjective probabilities in order to determine the probability that the defendant committed the assassination: first, the initial subjective probability that the defendant killed the victim [this probability is symbolized as P(K)]; second, the in-

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43 If, as stipulated, the car was going west of point X at speeds exceeding 70 miles per hour for 15 minutes, to determine the distance the car traveled one would take $15/60 \times 70$ for a result of 17.5 miles. Because the chase terminated at 1:15 p.m. and the defendant arrived in his office at 1:20 p.m., in order for both stories to be true, and for the defendant to be guilty, defendant would have to have driven 22.5 miles from the point at which the chase terminated to his office five miles east of point X. In order to calculate the speed at which the defendant had to travel one may use the following formula:

$$22.5 = 5 \cdot Sp, \quad \frac{22.5}{60}$$

where 22.5 is the distance hypothetically traveled, $5/60$ is the portion of an hour which was taken up in travel, the symbol "\times" means "multiplied by," and $Sp$ is the hypothetical speed in miles per hour. Without going through all steps of solving the equation, the result is 270 miles per hour.

44 This is the probability that the defendant killed the victim based on unquantified
itial subjective probability that the defendant did not kill the victim \[P(\text{not-K})\]; third, the initial subjective probability that the testimony that the defendant confessed (the "confession testimony") is true \[P(\text{Ad})\]; fourth, the initial subjective probability that the confession testimony is false \[P(\text{not-Ad})\]; fifth, the initial subjective probability that the car chase story is true \[P(\text{Cr})\]; sixth, the subjective probability that the car chase story is true if the defendant killed the victim \[P(\text{Cr}|K)\] — that is, the probability that Cr is true if K is true; seventh, the subjective probability that the car chase story is true if the defendant did not kill the victim \[P(\text{Cr}|\text{not-K})\]; eighth, the subjective probability that the confession testimony is true if the car chase story is true \[P(\text{Ad}|\text{Cr})\]; ninth, the subjective probability that the confession testimony is true if the defendant killed the victim and the car chase story is true \[P(\text{Ad} | (K \& \text{Cr}))\]; tenth, the subjective probability that the car chase story is true if the confession testimony is false \[P(\text{Cr}|\text{not-Ad})\].

The formula for the use of Bayes' Theorem, given this evidence, to determine the probability that defendant killed the victim, controlling for the dependency among the probabilities is:

The proof of this formula is based on Tribe, supra note 2. In the statement of the formula "&" represents joint probability. For example, \[P(K \& Cr \& Ad)\] is the probability that K & Cr & Ad all are true. The symbol "•" means "multiplied by." The proof begins with two basic postulates, which will be referred to as Formula 1 and Formula 2. Formula 1 and Formula 2 are:

\[
(1) \quad P(A \& B) = P(B) \cdot P(A|B) \\
(2) \quad P(B \& A) = P(A) \cdot P(B|A) \\
(3) \quad P(A \& B) = P(B \& A) \\
(4) \quad P(B) \cdot P(A|B) = P(A) \cdot P(B|A) \\
(5) \quad \frac{1}{P(A)} \cdot P(B) \cdot P(A|B) = \frac{1}{P(A)} \cdot P(A) \cdot P(B|A) \\
(6) \quad \frac{1}{P(A)} \cdot P(A) \cdot P(B|A) = \frac{1}{P(A)} \cdot P(B) \cdot P(A|B)
\]

Formula 2
Restatement of Formula 2
Identity
Step (3) with substitution according to steps (1) and (2)
Multiplication by equivalents
Equivalency
\[
P(K \& Cr \& Ad) = \frac{P(Ad|K \& Cr)}{P(Ad|Cr)} \\
\]

\[
P(Cr|K) \cdot \frac{P(K) \cdot P(Cr|K) + [P(\neg K) \cdot P(Cr|\neg K)]}{P(Ad) \cdot P(Cr|Ad) + [P(\neg Ad) \cdot P(Cr|\neg Ad)]} \cdot \frac{P(K)}{P(Cr|Ad)} \\
\]

(7) \[
P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)} \\
\]

(8) \[
P(A) = P(A \& B) + P(A \& \neg B) \\
\]

(9) \[
P(A \& \neg B) = P(\neg B) \cdot P(A|\neg B) \\
\]

(10) \[
P(A) = [P(B) \cdot P(A|B)] + [P(\neg B) \cdot P(A|\neg B)] \\
\]

(11) \[
P(B|A) = \frac{P(B) \cdot P(A|B)}{P(B) \cdot P(A|B) + [P(\neg B) \cdot P(A|\neg B)]} \\
\]

Step (11) is Bayes’ Theorem. The above proof is drawn from Tribe, supra note 2, at 1351-52 & n.72, and lays the ground for the following proof of the formula in the text.

(12) \[
P(K|Cr) = \frac{P(K) \cdot P(Cr|K)}{P(K) \cdot P(Cr|K) + [P(\neg K) \cdot P(Cr|\neg K)]} \\
\]

This yields the formula for calculating the probability that the defendant killed the victim if the car chase story is true. To determine the formula for the probability that the defendant killed the victim if both stories are true, one would do the following:

(13) \[
P(K|(Cr \& Ad)) = \frac{P(K) \cdot P((Cr \& Ad)|K)}{P(Cr \& Ad)} \\
\]

(14) \[
P(K|(Cr \& Ad)) = \frac{P(K \& Cr \& Ad)}{P(Cr \& Ad)} \\
\]

(15) \[
P(K|(Cr \& Ad)) = \frac{P(K \& Cr) \cdot P(Ad|(K \& Cr))}{P(Cr \& Ad)} \\
\]

(16) \[
P(K|(Cr \& Ad)) = \frac{P(Cr) \cdot P(K|Cr) \cdot P(Ad|(K \& Cr))}{P(Cr) \cdot P(Ad|Cr)} \\
\]

(17) \[
P(K|(Cr \& Ad)) = \frac{P(Ad|(K \& Cr)) \cdot P(K|Cr) \cdot P(Cr)}{P(Ad|Cr) \cdot P(Cr)} \\
\]

(18) \[
P(K|(Cr \& Ad)) = \frac{P(Ad|(K \& Cr)) \cdot P(K|Cr)}{P(Ad|Cr)} \\
\]

(19) \[
P(K|Cr) = \frac{P(Cr|K) \cdot P(K)}{P(K) \cdot P(Cr|K) + [P(\neg K) \cdot P(Cr|\neg K)]} \\
\]

(20) \[
P(K|(Cr \& Ad)) = \frac{P(Ad|(K \& Cr)) \cdot P(Cr|K)}{P(Ad|Cr) \cdot P(Cr|K)} \\
\]

Completion of multiplication from step (6) 

Step (7) with substitution according to steps (1) and (9) 

Step (7) with substitution for P(A) according to step (10) 

Restatement of formula in step (11) 

Restatement of formula in step (7) 

Step (13) with substitution according to Formula 2 

Step (14) with substitution according to Formula 2 

Step (15) with substitution according to Formula 2 

Commutativity 

Reduction to lowest common denominator 

Commative restatement of formula in step (12) 

Step (18) with substitution for P(K|Cr) according to step (19)
This formula and the subjective probability estimates required by it involve a number of extremely subtle interpretations of Bayes' Theorem. The application of Bayesian theory might even be so complicated as to be of no practical use.

Yet, given that the interpretation of one symbol for an inference in

The formula for $P(K \& Cr \& Ad)$ is the formula ultimately to be derived here given that one may rely only on evidence in the record in determining whether the defendant killed the victim. From Formula 2 it follows that $P(K \& Cr \& Ad) = P(Cr \& Ad) \cdot P(K|Cr \& Ad)$. If either $P(Cr)$ or $P(Ad)$ is equal to 1.0, then $P(Cr \& Ad)$ would be equal to $P(Ad)$ or $P(Cr)$, respectively. However, if neither $P(Ad)$ nor $P(Cr)$ is 1.0, then one also must know the formula for calculating $P(Cr \& Ad)$. Professor Tribe avoided this problem by assuming that his analogs were true. Tribe, supra note 2, at 1367. To determine $P(Cr \& Ad)$:

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\[
(21) \quad P(Cr \& Ad) = \frac{P(Cr) \cdot P(Ad|Cr)}{P(Ad) \cdot P(Cr|Ad) + [P(not-Ad) \cdot P(Cr|not-Ad)]}.
\]

2. Professor Tribe avoided this problem by assuming that his analogs were true. Tribe, supra note 2, at 1367.

3. To determine $P(Cr \& Ad)$:

\[
(22) \quad P(Cr) \cdot P(Ad|Cr) = P(Cr \& Ad).
\]

4. To determine $P(Cr \& Ad)$:

\[
(23) \quad P(Ad|Cr) = \frac{P(Cr \& Ad)}{P(Cr)}.
\]

5. To determine $P(Cr \& Ad)$:

\[
(24) \quad P(Cr \& Ad) = \frac{P(Cr|Ad) \cdot P(Ad|Cr)}{P(Ad) \cdot P(Cr|Ad) + [P(not-Ad) \cdot P(Cr|not-Ad)]}.
\]

6. The result in the text is obtained by changing the order of $P(K)$ and its immediate successor according to the principle of commutativity of multiplication.

7. For example, the probability that the confession testimony is true, given that the car chase testimony is true, $P(Ad|Cr)$, could be quite hard for a layman to distinguish from the probability that the confession testimony is true if the defendant killed the victim and the car chase testimony is true. Nevertheless, the distinction is very important.

8. The formula would require, in the case given, on only the evidence necessary for a prior probability $P(K)$ and the testimony as to the car chase story and the confession story, 11 subjective probability estimates, nine multiplications, three divisions, and two additions, at least some of which would involve working with some rather small decimal values. Further, the court would not only have to interpret the formula itself, but would have to communicate that interpretation to the jury. The arithmetical objection might be mitigated by use of a computer, though this possibly would dehumanize the trial process. Tribe, supra note 2, at 1375-77.
subjective probability theory may depend on the interpretation of other symbols, that every piece of evidence in a trial must tend to "make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence," and that these inferences about "facts . . . of consequence" would necessarily be related to the ultimate facts in the case and thus derivatively related to each other, use of these formulae adapted to fit cases with even more data on which inferences might be made is unavoidable. Because of the complexity of these formulae in the assassination example, with relatively few inferences necessary, any attempt to systematize factfinding in a normal case in a way that is consistent with the rigors of Bayes' Theorem must be a highly complex and problematical effort.

**Two Problems With Prior Probabilities**

Two difficulties with the use of Bayes' Theorem by factfinders are related to basic assumptions needed for the coherence of Bayesian theory. The first, often called the problem of "prior probabilities," results from the interpretation of Bayes' Theorem necessary for the theorem's application at trial; it requires that at least one probability be established based on testimony at trial without reference to the sufficiency of evidence on which to base that judgment or to the soundness of inferences on which the judgment is based.

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49 See D. Hofstadter, Gödel, Escher, Bach 102 (1979) (terms [or variables] reinterpreted, i.e., redefined, to make theorems compatible with other systems).

50 Fed. R. Evid. 401. The quotation of a definition of relevance which includes the word "probable" should not be taken as a concession that a datum's effect on a subjective probability estimate is a sufficient condition for the relevance of that datum for evidence purposes. See, e.g., notes 191-93 & accompanying text infra. The point here is that if one takes the reference to "probable" to include subjective probability, determinations attempting to control for the dependency of inferences at the very least become extremely complex. Yet one must test to control for dependency. Tribe, supra note 2, at 1359-61.

51 Of course, should there be no interpretation which avoids a contradiction, or no interpretation which avoids a contradiction and yet is linguistically cogent, an effort at Bayesian systematization goes for naught. M. Kline, supra note 1, at 95.

52 Brilmayer & Kornhauser, supra note 3, at 144. Bayesians have argued lack of particularized evidence should result in a downward adjustment of the estimate of subjective probability for the inference which that evidence supports, at least when there is not an adequate explanation for the failure to produce particularized evidence. E.g., Kaye, Res Ipsa, supra note 3, at 1474-79. See notes 80-87 infra for a discussion of the distributive justice implications of this view, that is, the issue posed when there is no evidence which can be produced. For a discussion of the extent to which uncertainty can be accounted for quantitatively, see note 29 supra; notes 61-66 & 177 & accompanying text infra. This argument might be taken to imply that prior probabilities should be similarly adjusted when based on paucity evidence. Yet it appears that advocates of the use of Bayes' Theorem as a factfinding guide have no response to Brilmayer and Kornhauser's argument that the principle of indifference, or Laplacean principle of insufficient reason, requires that in the event the decisionmaker has no information on which of a number of alternatives N is
Suppose that the only witness to an auto accident (the parties to the
accident are comatose due to injuries caused by the accident) is the
Catholic archbishop. Plaintiff’s guardian calls the archbishop to testify
that the defendant was exceeding the speed limit, that the street was
icy, and that the defendant’s car went through two 360-degree skids before
colliding with the plaintiff’s car. The archbishop further testifies that the
plaintiff’s car was parked in a legal parking place within six inches of
the curb. On cross-examination, much to the chagrin of defendant’s counsel,
the archbishop demonstrates that he has 20-10 vision and testifies that
he was a policeman with considerable experience in high-speed driving
and accident investigation before he entered the seminary.

Juror J, who has never driven an automobile, and has considerable diffi-
culty estimating the speeds of taxis or buses in which he has ridden,
concludes the archbishop’s testimony is somewhat unreliable because no
one can estimate the speed of automobiles with complete confidence. J’s
estimate of the prior probability of negligence with respect to the speed
of defendant’s automobile should be lower than the estimates of those
who considered the archbishop’s testimony reliable.\(^{43}\) J would not, strict-
ly speaking, be dealing with an estimation of the prior probability based

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\(^{43}\) As to the nature and effect of such a modification, see Brilmayer & Kornhauser, supra note 3, at 146. Basically, such an adjustment should reduce J’s estimate of the subjective probability that the archbishop’s testimony is true without reducing the subjective probability that it is false. See id. Contrast the discussion in note 65 infra of quantum logic’s recognition of the limits of symbols. To the extent such modifications take place, issues of insufficiency of evidence are raised. See Brilmayer & Kornhauser, supra note 3, at 146. See generally notes 167-93 & accompanying text infra.
on the evidence submitted at trial. Rather, if J used his own judgment in making subjective probability estimates, J would be reaching conclusions based on testimony received at trial and on his own rules of inference; in other words, based on rules of inference derived by J from his observations outside the courtroom, J would reach conclusions about the weight of evidence and inferences derived from that evidence.

Jurors are expected to bring their own experience to reasoning about the truth and falsity of testimony and, more importantly for the purposes of this article, to use their experience in determining the degree to which a piece of evidence entails the truth of one ultimate fact over another. Therefore, it is only the desire for rigorous analysis which motivates exponents of the use of Bayes' Theorem to ask the jury to re-examine the validity of the rules on which it bases its inferences from testimony to ultimate facts, and the reliability of the experience on which it bases its inferences. Were J provided with appropriate guidelines for this examination, then presumably J would be able to assess precisely the credibility of the archbishop's testimony. The point here is not that Bayesian theory is contrary to the typical practice in judicial instructions. Judges in most cases refrain from ordering the jury rigorously to re-examine its experience and the rules of inference derived therefrom, but Bayesian theory discourages re-examination of those basic assumptions. It may discourage re-examination of the basis of conclusions which jurors have drawn from their experience—a re-examination which is implicitly, if not explicitly, required by the rationale for use of Bayesian theory and is a desirable re-examination in which conscientious jurors might otherwise

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54 Otherwise, for example, it would make no sense for the United States Supreme Court to require, as it did in Ross v. Bernhard, 396 U.S. 531, 538 n.10 (1970), that in deciding whether an issue is required to be tried by a jury under the seventh amendment, the federal courts should consider inter alia "the practical abilities and limitations of juries."

55 See G. HARMAN, supra note 15, at 189-94 (concluding that remembered data must be considered to be reinferred for coherent account of inductive inference).

56 See, e.g., United States v. Gleason, 616 F.2d 2, 15 (2d Cir. 1979) (comments in judge's instructions to jury adverse to witnesses' credibility should be accompanied by statement that jury could well find otherwise, in whole or in part); United States v. Anton, 597 F.2d 371 (3d Cir. 1979) (holding trial judge's statements that witness was not credible reversible error; court reasoned that such statements rarely should be made).

57 Professor Tribe refers to this problem as the "dwarfing of soft variables." Tribe, supra note 2, at 1361-65. There is a marked temptation, once a subjective probability estimate is quantified, to ignore factors which are less quantifiable (as are the jurors' doubts about credibility here, see note 53 supra) and rely on the quantified factors. Professors Ellman & Kaye recognize the problem but argue that the chart approach can account for it. Ellman & Kaye, supra note 5, at 1156. As pointed out in note 52 supra, the chart invites the formation of a prior probability by demonstrating the effect of various prior probabilities on other quantified evidence. It does not even attempt to alleviate the difficulty of estimating probabilities accurately.

58 See notes 43-51 & 55 & accompanying text supra.
engage. Nor does probability theory provide a point at which the re-examination process should begin or end.

An exponent of Bayesian theory might argue that one would naturally discount the effect of one's own prejudices which are weakly founded by adjusting the subjective probability to take account of those prejudices. That is, if $J$, on the basis of testimony and $J$'s inferences from his experience, would assign a probability of guilt of .9, then that probability could be lowered, by $J$, to account for his prejudices, to, say, .6. The problem is that, if one is discounting for the uncertain wisdom of relying on one's own experiences, it is not necessarily true that the probabilities of guilt and innocence given his experience will equal 1.0. Because $J$ is not aware of the value of the inference to an unprejudiced juror, $J$ will not be precise in allowing for that prejudice. This imprecision could result in the sum of the probabilities of guilt and innocence being either greater or less than 1.0. If $J$'s subjective probabilities do not equal 1.0, then the Bayesian system fails because the sum of the mutually inconsistent and exhaustive probabilities does not equal 1.0, which is required under the

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See note 55 supra.

Non-Bayesian analysis admittedly provides no such point. But if a rigorous examination of one's inferences is a rationale for a factfinding model, then in order to ferret out the erroneous bases on which inferences might be made, for instance on one's poorly remembered past experiences, a re-examination of the degree of conditionality of these inferences is called for. Such a re-examination would be analogous to the conditionality test in the assassination case, see text accompanying 43 supra. The central difficulty here is selecting and segregating those inferences to be re-examined. There seems to be no way short of a study of all the variables which jurors employ in determining cases, coupled with a thoroughgoing prescription as to how jurors should systematize their prior experience in light of the tendency to use these variables. Such an attempt might be operationally impossible and in any event would far exceed, for example, the complexity of Professor Loftus' work on the accuracy of eyewitness testimony, E. LOFTUS, EYEWITNESS TESTIMONY (1979), or Professors Kalven & Zeisel's study of the jury function, H. KALVEN AND H. ZEISEL, THE AMERICAN JURY (1966). For further background on the difficulty of similar modeling, see Stroud, Inference, Belief, and Understanding, 89 MIND 179 (1979). The cases on the limits of the trial judge's authority even to comment on the evidence to the jury strongly imply that prescription of the way the jurors are to use their prior experience would exceed the judge's power, at least as currently conceived. E.g., Ah Lou Koa v. American Export Isbrandtsen Lines, Inc., 513 F.2d 261, 263-64 (2d Cir. 1975); authorities cited note 56 supra.

Cf., e.g., Ellman & Kaye, supra note 5, at 1159-61; Kaye, Gatecrasher, supra note 29, at 106-08 (arguing that one naturally discounts for uncertainty in the case of inadequate evidence to support an inference). The point seems to have originated with Professor Tribe. Tribe, supra note 2, at 1349-50.

Cf. Brilmayer & Kornhauser, supra note 3, at 136 n.73 (whether Bayesian methods are more accurate than "intuitive" methods is untestable in part because of difficulty in determining whether any verdict is correct). See also the discussion of Kaye's opposing view in note 29 supra.

As Bertrand Russell pointed out, the accurate quantification of uncertainty is seldom possible. B. RUSSELL, supra note 22, at 395. Problems of quantifying uncertainty should compound rather than alleviate the difficulty of making any accurate subjective probability estimate when the factfinder believes his estimate, made before adjusting for uncertainty, to be inaccurate.
postulates of the Bayesian system—that is, it does not follow the principle of additivity. One could force the subjective probabilities to assume a semblance of order by merely subtracting the probability of the first alternative assessed from 1.0 and assigning the result of the subtraction as the probability of the second alternative. For example, if the probability of guilt is .7, then the probability of innocence is 1.0 minus .7, or .3. But this assumes one has only two choices, thereby excluding one's option to refrain from deciding, and also places most, if not all, of the burden of uncertainty on the first alternative chosen because the juror will, through the subtraction, discount the first estimate for uncertainty and not have a similar opportunity to discount the result of the subtraction.

These problems of additivity and of discouraging sufficient rigor in testing "background conclusions" derived from the juror's own experience are common to all the estimates required for the use of Bayesian theory. These problems are only discussed at this point because of ease of explanation with probabilities that are nonconditional, for example, P(A), as opposed to conditional probabilities, for example, P(A|B). An issue which is equally important in considering the applicability of Bayesian theory is that of distributive justice. A return to the blasting cap case is illustrative.

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See notes 21-23 & accompanying text supra; note 46 supra (Formula 1).

The decision made here is that the evidence is inadequate for decisionmaking—a conclusion for which the legal system provides, but Bayesian theory does not. See note 52 supra; notes 176-77 & accompanying text infra. Gary Zukav has given an example of quantum logic's recognition that either/or conceptualization of a situation rests on a fallacious assumption that experience is bound by the same rules as symbols:

During the Lebanese Civil War, a story goes, a visiting American was stopped by a group of masked gunmen. One wrong word could cost him his life. "Are you Christian or Moslem?" they asked.

"I am a tourist!" he cried.

G. ZUKAV, THE DANCING WU LI MASTERS 286 (Morrow Quill ed. 1979). Bayesians should be similarly aware of false dichotomies and of the limits of symbols.

Professor Kaye argues that the odds-making model for fixing subjective probabilities in fact produces additive estimates, that is, estimates such that P(A) + P(not-A) = 1. Kaye, Laws, supra note 3, at 43 n.32. His argument is that an odds-maker who does not establish additive odds will soon alter his odds so that they are additive. Otherwise, the odds-maker will always lose money on the bets he accepts. This argument is not sufficient, however. Jurors who set nonadditive odds would not lose any money, so there would not be an economic incentive to avoid nonadditivity. While a program to educate jurors might produce jurors who estimated probabilities in an additive manner, such an educational program would have problems of its own. See notes 26-28 & accompanying text supra. Moreover, Professor Kaye, in this argument ignores what he concedes later in the same article: "[T]he juror can have the same confidence in the number produced by [Bayes' Theorem] as he does in the prior probability he estimates." Kaye, Laws, supra note 3, at 53. If probability estimates are to be altered merely to suit the axiom of additivity, without reference to the accuracy of the resulting estimates, confidence in the resulting estimates is misplaced.

See notes 61-66 & accompanying text supra.

See notes 54-60 & accompanying text supra.
The blasting cap hypothetical discussed earlier poses an issue of critical importance for subjective probability. Briefly, the hypothetical assumes that \( B \), a boy, is injured by a defective blasting cap of a definite type. \( M_1 \), a corporation, manufactures ninety-five percent of all blasting caps of this type with the remainder spread among three other manufacturers. Given the explosion of the blasting cap, there is no evidence of the identity of the manufacturer of the blasting cap. \( B \) sues \( M_1 \) on a theory of strict liability. The question is whether the evidence that \( M_1 \) manufactured ninety-five percent of the blasting caps would be a sufficient basis for a jury’s finding that \( M_1 \) manufactured the blasting cap which injured \( B \). One way in which scholars have sought to deal with similar problems is to hold that the evidence should be considered insufficient in order to encourage plaintiffs to seek and present particularized evidence—a reference to evidence other than statistical evidence. Such a response is inadequate in a case in which further evidence has literally, and without any proven blameworthy action on either \( B \)'s or \( M_1 \)'s part, gone up in smoke.

Relying on \( M_1 \)'s percentage of manufacture of all blasting caps as sufficient evidence that \( M_1 \) manufactured the particular blasting cap which injured \( B \) entails a number of necessary errors in the factfinding process. The foremost error is that even assuming statistics are truly probative in individual cases, the most probative statistic is not how many of the special blasting caps \( M_1 \) manufactured, but how many or what percentage of all the defective special blasting caps \( M_1 \) manufactured. Whether the court could accurately determine the percentage of defective blasting caps as opposed to the percentage of all blasting caps which \( M_1 \) manufac-

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tured depends on the extent to which all manufacturers of this type of blasting cap are forced to give evidence, the extent to which B is able to show that all products which could have caused B's injury were manufactured by the same process, and the ability of B or M₁ and the other manufacturers to prove percentage of defective manufacture.²⁴

Proof of manufacture of blasting caps with relevant defects to which the cap that injured B is sufficiently similar could be difficult in that first, there is the possibility of settlement of claims prior to trial, which would make ascertainment of defects in those situations difficult;²⁵ second, those defects which come to an attorney's attention will likely be only those defects which coincidentally cause injury or loss sufficient for the buyer to institute an action or to complain to the manufacturer; and third, the possibility of a product change and uncertainties about the content of inventories other than the blasting cap manufacturers' make the determination of the number of defective caps sold and market share of defective manufacturers extremely difficult. Although the disputes among the manufacturers pose no particular proof problem for B, the attendant delay and increased expense in B's lawsuit would give B an incentive to sue only M₁ as a defendant and to oppose the joinder of additional manufacturers as defendants.²⁶ The question is whether the law should be willing to allow B to establish M₁'s identity as the manufacturer of the defective blasting cap which injured B on the basis of M₁'s percentage manufacture of all blasting caps.

As indicated earlier, such a willingness would create imprecision to the extent M₁'s percentage of marketed blasting caps differs from M₁'s

²⁴ Sindell v. Abbott Laboratories, 26 Cal. 3d 594, 607 P.2d 924, 163 Cal. Rptr. 132 (1980), which would allow market share to be used to apportion liability among manufacturers in certain limited situations, is discussed at notes 153-82 infra, primarily because adequate understanding of the point which that case makes depends on development of the verification value model at notes 100-52 & accompanying text infra. It is only important here to note that Sindell is not based on any notion that mathematical logic provides, a priori or otherwise, a precise model of the correct result in a given case. 26 Cal. 3d at 612-13, 607 P.2d at 937-38, 163 Cal. Rptr. at 145-46.

²⁵ Such settlements also would be inadmissible to prove manufacture of a defective blasting cap. See Fed. R. Evid. 408.

²⁶ If the court is willing to adhere to a probabilistic rule of liability when X's subjective probability of manufacture of a particular defective blasting cap exceeds .5, then X may be held liable for all damages proximately caused by that defect and B may be able to avoid third party joinder of manufacturers other than M₁. The issue of M₁'s precise market share, once M₁ has a sufficient share to be held liable for all damages, is irrelevant to B. As long as M₁ is able to satisfy a judgment for B, the market shares of M₁'s competitors are irrelevant to B. B at least would have an argument, if M₁ attempted impleader of other manufacturers, that such an impleader would unduly complicate B's action. See 6 C. WRIGHT & A. MILLER, FEDERAL PRACTICE AND PROCEDURE § 1443 (1971). Even if such an argument should not prove successful, B could lay his case in a forum in which joinder is thwarted by the procedural rules of the forum, venue rules, or lack of personal jurisdiction. The same basic question is posed if M₁'s competitors at the time of the manufacture of the defective blasting cap cannot be joined because they are defunct or judgment-proof.
percentage of marketed defective blasting caps. Moreover, error is created over a number of cases to the extent $M_1$'s manufacturing percentage differs from 100 percent—in this case five percent. $M_1$'s percentage of the blasting cap sales may stem from the quality of $M_1$'s product. Thus, use of $M_1$'s percentage of manufacture (assuming it is above fifty percent both of all blasting caps and of defective blasting caps) poses additional problems not related to accuracy in $B$'s particular case because probabilistic logic would require that $M_1$ be held liable in those cases in which only $M_1$ is a defendant and in all similar cases. This forces $M_1$ to compensate plaintiffs for injuries that it did not cause, which increases the cost of $M_1$'s blasting caps. It is one thing to argue that the tort system should force $M_1$ to internalize the cost of injuries caused by its product and pass it on to consumers as a way of both compensating injured parties and increasing economic efficiency, but it is quite another to argue that $M_1$ should be forced to compensate all victims and raise its price to reflect costs which are not associated with its production. In effect such damages penalize $M_1$ for its large market share, even though the major reason for $M_1$'s market share may well be quality control.

Admissibility of market share evidence when only $M_1$ is joined would create a marked disincentive to the other manufacturers in the market to maintain quality control. With a product like blasting caps, which are by their nature likely to leave relatively little evidence behind, if liability can be placed exclusively on $M_1$, $M_1$'s competitors have little or no incentive to correct product defects; $M_1$ is in effect their insurer. No plaintiff who had a case for liability against $M_1$ would be likely to bother with the other manufacturers unless and until there was a problem in collecting a judgment from $M_1$.

A probabilistic rule thus not only would be on poor logical ground, but also would pose severe problems of distributive justice. “Distributive justice” here refers to Aristotle’s terminology for that kind of justice [which] is shown in the distribution of honour or money or such other possessions of the community as can be divided among its members. . . . It is admitted on all hands that in distributing shares justice must take some account of merit. By “merit,” however, people do not all mean the same thing.

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77 See note 73 & accompanying text supra.
78 See note 38 & accompanying text supra.
80 See generally R. Posner, supra note 79, § 6.2. The question whether torts should provide economic efficiency or loss spreading is not the central issue here. The central issue is that a Bayesian model can conceal issues that have little to do with the accuracy of a finding on whether A or not-A is true.
81 See note 79 supra.
The question is not one of mathematical logic but of the principle according to which assets and incentives are to be distributed. In other words, one must be careful to distinguish questions of the accuracy of a finding of fact from questions of the precise facts that must be found for-liability, for example, whether a policy of compensation or loss spreading in tort law requires that $M_1$ is liable in the blasting cap case based only on $M_1$'s market share, and from questions of the effects a particular procedural or evidentiary rule may have on extrajudicial conduct. One may choose to adopt an economic model or a noneconomic model as a guide to deciding whether to establish incentives or disincentives with regard to past and future behavior. It is these choices, whether one wishes to call them questions of distributive justice or of policy, which dogmatic application of Bayesian theory conceals.

A fairly common Bayesian argument may be stood on its head to illustrate this point. It is argued that the preponderance of the evidence test for the burden of persuasion in civil litigation supports the idea that legal factfinding conforms to a Bayesian model. If one assumes that errors in factfinding have equal disutility whether made in favor of one party or another, then the argument is that the preponderance test conforms to a probabilistic decisionmaking model for maximizing aggregate utility.\(^6\)

\(^6\)E.g., E. Posner, supra note 79. But see, e.g., Horwitz, Law and Economics: Science or Politics?, 8 Hofstra L. Rev. 905 (1980).

\(^5\)E.g., Starship 20:1-17.

\(^6\)For example, the attorney-client privilege excludes otherwise admissible evidence in order to protect the relationship between a client and his attorney. See C. McCormick, Handbook of the Law of Evidence § 87 (2d ed. E. Cleary 1972).

\(^6\)See, e.g., Lempert, supra note 3, at 1032-34; Kaye, Book Review, 89 Yale L.J. 601, 608 (1980). The most concise mathematical treatment is Note, A Probabilistic Analysis of the Doctrine of Mutuality of Collateral Estoppel, 76 Mich. L. Rev. 612, 622 n.31, (1978). By attempting to maximize utility, Bayesians attempt to minimize the disutility caused by errors of factfinding over all cases. As long as the assumption that an error for either side of a dispute has equal disutility holds, this is equivalent to attempting to minimize errors. Id. The problems posed when there is or seems to be a greater disutility in an error for one side or another are discussed in notes 87 & 194-210 & accompanying text infra. Bayesians to date have not argued that the utility of a finding should should be balanced in with the probability that the finding is true to determine whether the finding should be made. A utilitarian could argue that when $U(A)$ is the utility of $A$ and $P(A)$ is the probability of $A$, a factfinder should find in accordance with $A$ when $P(A) \cdot U(A)$ is greater than $P(\text{not-}A) \cdot U(\text{not-}A)$ and find not-$A$ where the latter is greater than the former. Even when jurors are required to appraise the social utility of conduct, for example, in deciding whether "reasonable" care has been exercised in a negligence case, e.g., United States v. Carroll Towing Co., 159 F.2d 169, 173 (2d Cir. 1947), the question of the social utility of the conduct is considered discrete from issues of factfinding. Lempert, supra note 3, at 1032-41. For the implications of a system which combines estimates of utility and probability, see Fisher, Truth as a Problem for Utilitarianism, 89 Mind 249 (1980). Robert Bartels argues that the reasonable doubt standard in criminal cases depends on the significance of the punishment that a conviction would entail. Bartels, Punishment and the Burden of Proof in Criminal Cases, 66 Iowa L. Rev. 899 (1981). This argument involves a consideration of the disutility of conviction to the accused and not to society in general. Compare the discussion in text accompanying notes 207-09 infra.
A change in the requirements of proof employed in enforcing a rule of tort law can cause the utility resulting from enforcement of that rule to vary from the utility that the rule was intended to secure. Assume, for example, that product liability is founded only on forcing manufacturers to internalize the damages caused by defects in products they manufacture. If $M_1$ is forced to compensate all plaintiffs comparable to $B$ in the blasting cap case, whether based on $M_1$'s share of the market or on $M_1$'s share of the manufacture of defective blasting caps, the balance of utility changes. $M_1$'s liability then is based on its size, which naturally discourages capture of a market. Plaintiffs similarly situated to $B$ become more financially secure as the law tends to operate as a mechanism for spreading their losses rather than as a facilitator of choices through the market. Whether or not such a trend in tort law is desirable, the assumption that use of a Bayesian system as opposed to an ordinary "intuitive" system is neutral in terms of distributive justice is erroneous.77

THE VERIFICATION VALUE STANDARD

An examination of some rules of evidence will show that verification value is preferable to Bayes' Theorem as a standard for measuring rules of evidence. Verification value is a concept derived here from those epistemological schools which hold that knowledge can only be derived through testing, either formal or informal.88 These schools hold that a proposition is true only to the extent it survives attempts to disprove

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77 Professor Kaplan noted the problem inherent in an assumption of equal utility of errors in tort law, given the trend to compensation as the model of tort law. Kaplan, supra note 3, at 1072. Another example of a possible use of Bayesian theory which would alter a balance of utility is a rule which changes settled expectations which in turn are required for meaningful planning to occur. See, e.g., H. Hart & A. Sacks, The Legal Process 4-6 (Tent. ed. 1958).

The burden of persuasion in criminal cases is greater than that in civil cases, and it is not difficult to see this as a result of a perception that an error in conviction is more grave than an error of acquittal. E.g., Kaplan, supra note 3, at 1073; Lempert, supra note 3, at 1038. This perception should not necessarily be taken as an indication that the reasonable doubt standard in criminal cases is any more closely correlated with probability than is the preponderance of the evidence standard in civil cases. To the extent Bayesian methods increase or decrease the prosecution's ease in obtaining convictions, they vary the balance of utility from that resulting from "intuitive" methods. The true issue is whether and why an increase or decrease in convictions is desirable. An additional issue relating to possible mathematical modeling in criminal cases is whether forcing or encouraging jurors to quantify the meaning of "a reasonable doubt" is permissible under the reasonable doubt standard. There is a very good argument that it is not. Tribe, supra note 3, at 1372-75. Advocates of the use of subjective probability could argue that the value of increased accuracy with the use of Bayes' Theorem outweighs whatever changes in the distributive justice consequences may result. As the claim of increased accuracy is dubious, this argument is insufficient. See notes 20-68 & accompanying text supra.

88 E.g., G. Harman, supra note 15; K. Popper, supra note 13.
it.\textsuperscript{97} One defender of this view finds subjective probability theory objectionable on these grounds.\textsuperscript{99} Verification value, as used here, is an adaptation of this view to the demands of the trial process. One cannot insist, or even expect, that all the contentions of the parties at trial as to past events can be empirically tested. Nor should a court ignore previously recognized scientific doctrine merely on a party's proof.\textsuperscript{93} Bayesians might object that knowledge is not a necessary outcome of the trial process; rather, the standard of proof is normally the preponderance of the evidence.\textsuperscript{92} Philosophers of knowledge such as Professor Popper do not take knowledge to require that the proposition which is known is certainly and eternally true;\textsuperscript{93} rather, they distinguish tested and testable theories from subjective judgments.\textsuperscript{94}

As used here, the "verification value"\textsuperscript{95} of evidence is assessed when that evidence is submitted to the jury or taken under consideration by a judge. The verification value of evidence increases as the rigor, quality, and number of tests which the evidence has withstood increase, whether those tests were cross-examination or other tests conducted at the trial or were empirical tests conducted outside of court. Regardless of other tests, verification value becomes null when evidence is contradicted by propositions which the court is not prepared to abandon, or allow the jury to abandon, on a party's proof—that is, judicially noticeable

\textsuperscript{97} See K. Popper, supra note 13, at 108. Professor Harman suggests that "[o]ne may infer a conclusion only if one also infers that there is no undermining evidence one does not possess." G. Harman, supra note 15, at 151. By placing emphasis on the gathering of evidence in this manner, Professor Harman in effect requires attempts to disprove in order to gain knowledge.

\textsuperscript{99} K. Popper, supra note 13, at 189-96. Professor Harman, on the other hand, faults a probabilistic rule of acceptance because it does not allow for examination of intermediate conclusions. G. Harman, supra note 15, at 120-24. Given Harman's heavy reliance on intermediate inferences as a source of knowledge, see id. at 168-71, there is a close analogy to Popper's insistence that a hypothesis be falsifiable in order for it to be part of a proper empirical scientific system. K. Popper, supra note 13, at 40-41.


\textsuperscript{95} See generally C. McCormick, supra note 85, § 339.

\textsuperscript{92} K. Popper, supra note 13, at 111.

\textsuperscript{94} Id. at 44-48. Of course, the idea that theories must be testable creates some uncertainty, but as Popper points out:

[T]he fact that the tests cannot go on for ever does not clash with my demand that every scientific statement must be testable. For I do not demand that every scientific statement must have in fact been tested before it is accepted. I only demand that every such statement must be capable of being tested; or in other words, I refuse to accept the view that there are statements in science which we have, resignedly, to accept as true because it does not seem possible, for logical reasons, to test them.

\textit{Id.} at 47-48 (emphasis in original).

\textsuperscript{96} This terminology is derived from logic, which includes the concept of "truth-value."

A three-valued logic has three truth-values, "truth, falsity and something intermediate." W. Quine, Philosophy of Logic 84 (1970).
propositions. Specifically, when an inference has survived some testing at trial, a court will not allow that inference to be maintained when it is contrary to general knowledge or to "sources whose accuracy cannot reasonably be questioned." This last standard acts as a means of testing evidence against sources other than those offered at trial. The emphasis on testing in the verification value criterion is in marked contrast to the untested mathematical systematization which Bayesians advocate.

Given that Bayesian analysis has certain flaws, one might wonder if it nevertheless conforms to the rules of evidence. The distinction between habit and character evidence, namely that evidence of a habit to prove conduct is admissible, while evidence of character to prove conduct is generally inadmissible, shows evidence law effectively minimizing the
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distributive justice and epistemological problems which Bayesian analysis raises. The difference between character and habit is the repeated and consistent practice of the habit as opposed to the occasional practice which is called character. This distinction is closely analogous to the distinction Professor Popper has drawn between probability and degree of confirmation or corroboration. He sees subjective probability as distinct from the degree of confirmation because subjective probabilities cannot be verified by testing. According to this view, confirmation is furnished

Other Acts of Sexual Misbehavior and Perversion as Evidence in Prosecutions for Sexual Offenses, 6 Am. L. Rev. 212, 221-31 (1965). Such holdings pose a severe problem for a character-habit dichotomy, in that the cases rely on a predisposition per se. Given that they seem to view all unnatural sex crimes as similar and that the rule shows no sign of invasion into other areas, the rule can be considered a warping of the modus operandi or common plan rules to fit a highly emotional situation. Id. at 234-35. When it is kept in mind that most jurisdictions still at least formally require that the past unnatural act has been committed with the same person, the admissibility of sex-crime-propensity evidence in certain jurisdictions can be seen as merely anomalous. See R. LEMPERT & S. SALTZBURG, A MODERN APPROACH TO EVIDENCE 220-21 (1977).

Professors Lempert and Saltzburg argue that evidentiary rules, of which the rule generally excluding character evidence is an example, which appear to exclude relevant evidence, exclude evidence which is seldom probative. Id. at 181-246; Lempert, supra note 3, at 1031. Professor Lempert notes that, even when such evidence would appear relevant to a contention, it is in circumstances in which much more specific evidence about facts in issue is normally available and should be used. If such more specific evidence is not offered, the factfinder should find against the contention. Id. at 1031 n.35, 1049. This analysis does not seem premised on any probabilistic theory of evidence. Thus, it is a laudable attempt to alter a Bayesian model (which Lempert presents elsewhere in the cited works) to correct for some shortcoming of probability theory. It has a defect, however, in that it does not explicitly grapple with the situation in which the lack of more particularized evidence is adequately explained—a distributive justice problem. See notes 71-87 & accompanying text supra; notes 171-82 & accompanying text infra.

Professor Lempert also attempts to square the “relevance rules,” as he refers to those rules exemplified by article IV of the Federal Rules of Evidence, with Bayesian theory in another manner. He argues that rules excluding evidence which would be excluded as “prejudicial” under the ordinary evidence lexicon are justified according to the Bayesian model because of problems in estimating the probability value of such evidence. Lempert, supra note 3, at 1027-30. According to his argument, other evidence is excluded by the relevance rules because so little is known about the relationship between the evidence and the parties’ contentions that estimation is similarly difficult. Id. at 1029. To the extent Professor Lempert wishes to use this argument for more than showing that Bayesian theory is roughly analogous to evidence law, it fails. Quantification of subjective probabilities is so unreliable in and of itself that the application of subjective probability theory at trial is unworkable. See notes 20-29 & accompanying text supra. Accordingly, marginal changes in regard to some quantifications will do little good. There may also be a problem with Lempert’s argument in that the theory of indifference could be read as requiring a subjective probability estimate even though no relevant evidence has been adduced. See note 52 supra.

103 Compare 1 J. WIGMORE ON EVIDENCE § 92 (3d ed. 1940) with id. § 65. As an illustration, under new codes patterned on the Federal Rules of Evidence, character may be proven by use of specific instances of conduct. Fed. R. Evid. 405(b). See also Kuhns, The Propensity to Misunderstand the Character of Specific Acts Evidence, 66 Iowa L. Rev. 777 (1981).

104 K. POPPER, supra note 13, at 387 app.

105 See id. at 387-89. Professor Popper also argues that both sorts of probability are not
only by testing the accuracy of a hypothesis by experience. Furthermore, "a hypothesis may be very probable simply because it tells us nothing, or very little. A high degree of probability is therefore not an indication of [the acceptability of a hypothesis]—it may be merely a symptom of low informative content."

An illustration of the difference between probability and corroboration is an observation of an intersection in a residential neighborhood. If the observation were conducted from 1 a.m. to 2 a.m. on Monday and the only car observed were a green Lamborghini, one would be required to conclude, on a strictly probabilistic rule of acceptance, that the only vehicles which ever passed through that intersection were green Lamborghinis. Therein lies the distinction between high probability with low informative content and corroboration—the fact that no one would be inclined to accept the result of so limited a test as conclusive.

In the case of character evidence, the evidence is basically an induction made by the witness based on a number of experiences with or recollections about the subject of the testimony. Some of these experiences or recollections may be inconsistent with the characteristic which the evidence is offered to prove. Habit evidence differs from character evidence in that the former is evidence of the "invariability" of the subject's conduct through a number of situations identical in all material respects to the situation at issue. Thus, the requirements for the admissibility of habit evidence parallel Popper's theory of corroboration equivalent to corroboration in that they do not state the "degree to which a statement is supported by a statement y." Id. at 391.

See id. at 399.

See id. (emphasis in original).

See notes 14-18 & accompanying text supra.

E.g., Frase v. Henry, 444 F.2d 1228, 1232 (10th Cir. 1971) ("'Character' is a generalized description of one's disposition in respect to a general trait such as honesty, temperance or carefulness'; court held habit, on the other hand, to exist where deceased "showed a regular practice at meeting a particular situation with a specified type of conduct").

Baldridge v. Matthews, 378 Pa. 566, 106 A.2d 809 (1954). Concededly, courts may have trouble deciding whether a practice is a habit or merely a part of the subject's character. For instance, the number of repetitions of the situation could have been sufficiently few, the set of circumstances could be too imprecisely described, or the court could decide that the supposed "habit" is too much like the subject's normal conduct to be isolated as a habit. See generally J. MAGUIRE, J. WEINSTEIN, J. CHADBourn & J. MANSFIELD, CASES AND MATERIALS ON EVIDENCE 977-80 (6th ed. 1973). Although the distinction is fuzzy at the edges, the courts still accept it. Fed. R. Evid. 404, 406; Lewan, The Rationale of Habit Evidence, 16 SYRACUSE L. REV. 39, 49-51 (1964) (arguing that extent of repetition should be admissible to check abuses of overstated testimony, but not on probabilistic rationale). "Invariable regularity" is Wigmore's standard. 1 J. WIGMORE, supra note 103, § 92, at 520. The federal rules merely use the word "habit" for persons and "routine practice" for organizations. Fed. R. Evid. 406. The text at note 116 infra qualifies the objective probability analogous to habit evidence as near 1.0. This qualification is only because it is doubtful that one's conduct on every similar occasion can be proven in every case.

See notes 104-07 & accompanying text supra.
in requiring that the habit be examined against a standard and in the
degree to which the survival of the hypothesis depends upon how many
times the practice has been repeated. Although requirements of the trial
process are somewhat different from those of the laboratory, the concept
of verification value as used herein is the general equivalent of Popper's
notion of corroboration for evidence law, differing only in acknowledging
first, that certainty in evidence or conclusions cannot be expected; and
second, that the court does not have time to re-examine any generally
accepted or verifiable theory which a party might wish to question.118 The
requirements for admissibility of habit evidence control for verification
value by forcing proponents of such evidence to show invariable repeti-
tion and to explain behaviors inconsistent with the proponent's conten-
tion. This process reduces the problems of distributive justice119 and
epistemology which broad-scale admissibility of character evidence would
present.120

The distinction between objective and subjective probabilities is not
identical to the habit-character distinction. Character testimony involves
more observation, specific or general, than a subjective probability
estimate would require.115 Objective probability statements will normally
result in a probability of much less than the near 1.0 which habit evidence
requires.116 Yet, objective probability and habit evidence do differ markedly
from subjective probability and character. The difference is one of kind
rather than degree. Habit evidence and objective probability117 both re-

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112 As to testing against prevailing scientific theories, see notes 131-36 & accompanying
text infra.

113 The problem here is one of distributive justice insofar as admissibility of character
evidence may either cause X to be convicted or liable, when Y is the culpable party, or
discourage X from rehabilitation. See notes 82-87 & accompanying text supra.

114 A severe example of the effect of admissibility of character evidence is the use of
prior episodes of unchaste behavior by an alleged victim in a rape case to show increased
likelihood of the victim's consent. See Letwin, "Unchaste Character", Ideology, and the Califor-
nia Rape Evidence Laws, 54 S. Cal. L. Rev. 35, 42, 52 (1980). An assumption that because
a woman consented to intercourse with a particular man in a particular set of circumstances,
she would be more likely to consent to intercourse with any man under any circumstances
(which a general rule of admissibility would assume), is a classic example of assumption
of a pattern of conduct in a situation in which the sample is grossly inadequate. While
one might argue that admissibility of prior unchastity does not affect distributive justice
in that it results in a decreased number of convictions of the innocent, the argument would
ignore the fact that such a rule deters prosecution and tends to avoid conviction of the
guilty. Given that the probative value of prior unchastity in a rape case is generally minimal
and the difference in prosecution and ability to evade conviction is much greater, the balance
is heavily in favor of inadmissibility. The epistemological problems referred to are dis-
cussed at notes 115-31 & accompanying text infra.

115 Bayesian theory does not require observations establishing the accuracy of one's sub-
jective probability estimates prior to their application. See, e.g., Kaye, Laws, supra note
3, at 43 & n.32; note 52 & accompanying text supra.

116 See note 110 & accompanying text supra.

117 See generally K. POPPER, supra note 13, at 212.
quire an accounting for, and induction from, every instance, although that accounting may be erroneous. Subjective probability and character evidence require only an induction based on a portion of the total evidence, including a number of possible observations, on which an induction could be made. When a subjective probability estimate is made, one's facility for making the estimate, given the difficulties in testing the accuracy of one's estimates and the possible problems with their conformance to the requirement of Bayes' Theorem, is an extremely slender basis for inferring anything with confidence. Similarly, if character evidence were admissible to prove conforming conduct, one might properly testify that a subject has a given character even though a number of the witness' observations have been to the contrary or the witness has observed the subject too few times to testify that the subject has a habit. The verification value of character evidence of jurors' estimates of subjective probabilities is slight.

The two main areas in which character evidence is admissible to prove conduct are evidence of the character of defendant to raise a reasonable doubt of guilt and evidence of a witness' bad character for purposes of impeaching the witness. In its case-in-chief, the prosecution in a criminal case generally is prohibited from using character evidence to prove conduct. The purpose of the rule is to prevent confusion of issues, unfair surprise, and undue prejudice. The defendant in a criminal case

118 Kaye, Laws, supra note 3, at 53. Admittedly, an error or a number of errors of observation may be included in the events recorded in determining an objective probability. However, the number of observations on which objective probability figures are normally based tends to limit the effect of any one error by providing numerous opportunities to balance or discover it.

119 As to character, see notes 100-03 & 108-09 & accompanying text supra. As to subjective probability, see note 52 supra.

120 See notes 20-87 & accompanying text supra.

121 As an example, Compton v. Jay, 389 S.W.2d 639, 642-43 (Tex. 1965), holds that two prior convictions of driving while intoxicated are not sufficient to establish frequent or habitual drunkenness such as would be admissible to impeach the defendant on the issue of drunkenness or to show that the defendant was intoxicated at the time of the relevant accident, even when there was other evidence of intoxication. Two convictions of driving while intoxicated, especially if well publicized, however, could become a part of the defendant's reputation or of a witness' perception of that reputation. Testimony of reputation is admissible when evidence of character is admissible. E.g., Fed. R. Evid. 405(a).

122 E.g., Edington v. United States, 164 U.S. 361 (1896).

123 E.g., Fed. R. Evid. 608.

124 The prosecution nevertheless might be able to introduce evidence of past crimes committed by the defendant. For example, when the prior crime is an element of the crime charged, as under a habitual criminal statute, evidence of the prior crime is admissible. Michelson v. United States, 335 U.S. 469, 475 n.8 (1948), and authorities there cited. For other instances in which evidence of past crimes is admissible, see C. McCORMICK, supra note 85, § 190, at 447-54.

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may introduce evidence as to his character, not because it is particularly probative of his conduct, but because of a policy concomitant with the reasonable doubt standard favoring the admission of evidence which might tend to exculpate. Admissibility of evidence of a witness' character for purposes of impeachment is based on a slightly different analysis. Although relatively trivial in value, evidence as to the witness' character for truth and veracity is admissible to test the witness' testimony as part of the process of determining the verification value of the testimony. The credibility of testimony rests on the jury's assessment of the witness' qualities such as moral character. The habit-character distinction thus supports verification value as an important objective in evidence law.

Verification Value, Judicial Notice, and the Hearsay Rule

The concept of judicial notice and the hearsay rule pose potential problems for the verification value theory. At first blush, the practice of judicial notice of adjudicative facts might be seen as a major flaw in the argument that maximizing verification value is an important standard in formulation of rules of evidence. One might see the taking of notice of a matter, after which it is deemed concluded, as showing that evidence law does not necessarily have verification value restraints. The Federal Rules of Evidence, however, refer to noticeable facts as not being "subject to reasonable dispute" and to sources "whose accuracy cannot reasonably be questioned."
The facts of *Nicketta v. National Tea Co.* provide a basis for illustration of the distinction between judicially noticeable data and absolutely certain data. The *Nicketta* court held that the trial court was justified in dismissing plaintiff's complaint of breach of an implied warranty of fitness of pork after proper cooking. The complaint alleged that the plaintiffs contracted trichinosis from pork that was properly cooked. Both courts took judicial notice of the proposition that one cannot contract trichinosis from pork which is properly cooked.

If one gives the theory of natural selection any credence at all, it seems entirely possible that trichinae might evolve which are much less heat-sensitive than their forebears. By way of analogy, viruses and insects which are much less sensitive to certain antibiotics or chemicals have already developed. The *Nicketta* court noted that scientific "facts" are not immutable. The pivot on which *Nicketta* turns is not impossibility of dispute about the possible causes of trichinosis but, rather, general acceptance by scientists that trichinosis can only be contracted by eating pork which has lacked proper cooking or any refrigeration. When this reasoning is coupled with the fact that on summary judgment the plaintiffs' only contention to the contrary was that trichinosis could result from pork that was *merely* cooked, judicial notice may be seen as a test of the verification value of the evidence of the party opposing judicial notice against then-accepted scientific theories.

The rule against hearsay, in its design to increase cross-examination of judicially noticeable facts. See 1 J. Weinstein & M. Berger, *Weinstein's Evidence* ¶ 201 [03] (1980). The term is not meant to imply that certainty is required. Rather, the test is whether the matter is "beyond practical dispute." Id. at 201-31 (emphasis added). As Morgan, perhaps the strongest proponent of the "indisputable" school, conceded: "the judge and the parties may often reasonably be in doubt as to whether the matter is disputable; and whether the proposition involved is disputable may be the subject of dispute among reasonable men." 1 E. Morgan, *Basic Problems of Evidence* 9 (1961).

The hearsay rule has, of course, been under attack. E.g., Weinstein, *Probative Force of Hearsay*, 46 Iowa L. Rev. 331 (1961); Note, *The Theoretical Foundation of the Hearsay Rules*, 93 Harv. L. Rev. 1760 (1980). The criticism of the rule is either based on the idea that juries are perfectly capable of appreciating the possible defects in hearsay testimony, id. at 1792; see, e.g., Chadbourn, *Bentham and the Hearsay Rule—A Benthamite View of Rule 63(4)(c) of the Uniform Rules of Evidence*, 75 Harv. L. Rev. 932, 939 (1962), or that the rule and its exceptions are too woefully contrived for their purpose, see, e.g., Note, supra, at 1787-93. As to the first point, given the breadth of the exceptions to the rule, that evidence which is in fact excluded by the hearsay rule is excluded as a control over the jury's discretion. The point is that evidence which is so unreliable that it cannot come within any exception to the hearsay rule is not an adequate basis for a finding of fact by any juror, despite his discernment. Thus conceived, the rule is analogous to a directed verdict standard. Cf. Allen, *Structuring Jury Decisionmaking in Criminal Cases: A Unified Constitutional Approach to Evidentiary Devices*, 94 Harv. L. Rev. 321, 325 (1980) (arguing that functionally
of out-of-court statements,\textsuperscript{137} acts to increase verification value of evidence presented to the jury by securing testing of statements which could not otherwise be effectively tested. Verification value is increased by forcing the party offering the statement either to put the declarant on the stand or to forgo the evidence.

Exceptions to the hearsay rule are somewhat less easy to reconcile with a verification value maximization objective. The reliability of some evidence receivable under the exceptions seems extremely questionable. One might question whether evidence received under the spontaneous exclamation, or excited utterance, exception\textsuperscript{138} is very reliable.\textsuperscript{139} There may be special doubts as to the requirement that the declarant be under stress when the statement is made. Similar doubts arise about the requirement that a statement admissible under the present sense impression exception\textsuperscript{140} be made during or immediately following the declarant's perception of the event or condition described in the statement, at least when the event is stressful or exciting. It is well accepted in the psychological literature that stress may well have an adverse effect on the accuracy of a witness' perception.\textsuperscript{141} Psychological research indicates that jurors are not particularly good at controlling for this problem, especially when the situation at issue was a violent one.\textsuperscript{142} Given these difficulties, one might ask whether there is any basis to assume that spontaneous utterance or present sense impression evidence is more reliable than evidence untested against any standard.

The psychological theory on which the spontaneous exclamation rule was originally founded is that stress and excitement prevent the declarant from exercising his faculties to distort the truth to serve his own similar evidentiary devices should be subject to a similar constitutional standard). As far as this author is aware, no critic of the hearsay rule on this ground has ever shown a single specific instance when otherwise inadmissible hearsay would, absent the hearsay rule, be sufficient to support a verdict. The arguments that the existing rules are too wooden, or that a balancing test should be employed, are not particularly relevant to this article, assuming that verification value as well as probability is a criterion in determining the admissibility of the evidence. Two observations seem in order. First, as rules would undoubtedly develop over time as to the application of a balancing standard, and the existing exceptions to the hearsay themselves have developed through a balancing process, marginal changes in the exceptions would seem less wasteful in the long run. Second, the mode of balancing which the note cited above advocated, namely, quantifying the reliability of evidence, is highly questionable. See notes 20-29 & accompanying text supra.

\textsuperscript{137} See, e.g., J. Weinstein, Basic Problems of State and Federal Evidence, by Edmund M. Morgan 227 (5th ed. 1976).
\textsuperscript{138} E.g., Fed. R. Evid. 803(2).
\textsuperscript{139} Hutchins & Slesinger, Some Observations on the Law of Evidence, 28 Colum. L. Rev. 432 (1928).
\textsuperscript{140} E.g., Fed. R. Evid. 803(1).
\textsuperscript{141} See E. Loftus, supra note 60, at 33-36, 151-56.
\textsuperscript{142} Id. at 173-75.
self-interest. This "mentalism" psychological theory, even if an anachronism now, was generally accepted by the courts at the turn of the century. Continuation of rules allowing admission of statements made under stress is not due to uncritical acceptance of evidence offered at trial, but to reluctance to reexamine hoary legal rules, even though the rules are based on notions of refuted, or at least highly doubtful, scientific principle. The question is not whether particular exceptions to the hearsay rule undercut a verification value maximization policy because the exceptions were originally adopted on then-contemporary theories of psychological or physical reliability. The question is whether there is any reason for the continuance of these exceptions.

Verification Value and Distributive Justice

One other possible problem with verification value analysis remains, namely, whether and to what extent adherence to a maximum verification value criterion increases or decreases distributive justice problems. Careful adherence to a maximal verification value standard mitigates problems of distributive justice in two ways. In combination with an absence of probabilistic analysis, it forces the use and maximum discovery of particularized evidence and encourages the testing of that evidence to the maximum extent feasible. Particularization avoids the problem of severe prejudice which may arise, for instance, in Professor Tribe's blue bus hypothetical in which an alleged victim testifies she was run over by a blue bus and offers to show that X Bus Company operates ninety percent of the blue buses operating on that street. There is a great likelihood that such evidence would prejudice the jury in her favor on the questions of the reliability of the alleged victim's testimony. The more particularized the plaintiff's proof, the less likely there is to be a

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144 Hutchins & Slesinger, supra note 139, at 435.
145 1 S. Greenleaf, A Treatise on the Law of Evidence § 162g. (16th ed. Wigmore ed. 1899) and cases there cited.
146 See generally B. Cardozo, Growth of the Law, in Selected Writings 173 (M. Hall ed. 1947). Curiously enough, the present sense impression exception came about in part as a reaction to the idea, which was strongly pressed by Wigmore, that shock or excitement is a guarantor of veracity. See also Waltz, The Present Sense Impression Exception to the Rule Against Hearsay: Origins and Attributes, 66 Iowa L. Rev. 869, 875-76, 897 (1981).
147 See notes 73-87 & accompanying text supra (discussion of distributive justice issues which subjective probability poses).
148 One of Professor Tribe's major complaints is that use of probabilistic evidence would tend to discourage full use of discovery procedures. Tribe, supra note 2, at 1349-50.
149 Id. at 1340-41, 1346-47.
150 This is referred to by Professor Tribe as the "dwarfing of soft variables." Id. at 1361-65; see note 57 supra. See also notes 194-210 infra (discussion of prejudice).
broad gauge error in determining the facts in a particular case.

The major contribution of maximizing verification value is to present distributive justice issues clearly by avoiding probabilistic clouding of the issue. The real issue in terms of effective adjudication in the blasting cap case is not whether the blasting cap which injured the plaintiff was manufactured by one defendant or another because the manufacturer of the specific blasting cap which caused the injury is unascertainable. The important point is whether, and by what means, the plaintiff shall have compensation, assuming that the blasting cap was defective. The court is faced with the decision whether to assess damages to the class of manufacturers to which the defendant belongs and on what basis. Subjective probability has nothing particularly to do with the problem. Once the rule upon which liability is to be determined has been established, then statistics play a role.

An illustration of a rule which does allow reliance on probabilistic evidence, but as a supplement to a rule of compensation, rather than on a theory that it is required by mathematical logic, is that of Sindell v. Abbott Laboratories. Under Sindell, a plaintiff who was injured by the administration of a specific drug, DES, to her mother during pregnancy, but who cannot identify the manufacturer of that drug through no fault of her own, may nevertheless recover against all makers of that specific drug who used an identical formula and who sold the drug in the appropriate market. The plaintiff must join defendants who have manufactured a substantial percentage of the product marketed. The individual manufacturers have the burden of proving that they could not have made the particular dosage or dosages which injured the plaintiff. Liability

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111 Notes 30-33 & 69-81 & accompanying text supra.
113 26 Cal. 3d 588, 607 P.2d 924, 163 Cal. Rptr. 132 (1980).
114 Id. at 610-13, 607 P.2d at 936-37, 163 Cal. Rptr. at 144-45. Note there were no contentions of difference in quality control procedures. See 26 Cal. 3d at 612, 607 P.2d at 937, 163 Cal. Rptr. at 145. The court appears to hold that joined defendants can avail themselves of third party joinder or, if they are held liable, seek contribution from other manufacturers. Id. There is some question whether other states would be willing to recognize the Sindell theory. E.g., id. at 617, 607 P.2d at 940, 163 Cal. Rptr. at 148 (Richardson, J., dissenting); Ferringo v. Eli Lilly & Co., 175 N.J. Super. 551, 569-70, 420 A.2d 1305, 1314 (1980).
115 What is important for purposes of this article is not whether other states are prepared to accept a theory such as Sindell. There are instead two points: first, that Sindell faces the true issue—whether there shall be compensation even though evidence as to the precise tortfeasor is lacking—decides that issue, and then apportions damage in terms of the only arguably sound mathematical application available, by apportioning damages according to shares of the proper class (manufacturer of DES according to an identical formula rather than all manufacture of DES); and second, if the courts are unwilling to accept probabilistic methods in a case like Sindell, there would be little reason to suppose that Bayesian analysis would be acceptable.
116 26 Cal. 3d at 612, 607 P.2d at 937, 163 Cal. Rptr. at 145.
117 Id.
is determined by the proportion that each defendant's share of the appropriate market bears to the aggregate shares of the appropriate market of all joined defendants.167

The Sindell court's analysis is an improvement upon Bayesian analysis in numerous respects. Sindell makes explicit the problems of compensation, deterrence of product defects, and the lack of other available evidence as the prime determinants in the choice of a probabilistic scheme.168 The case does not purport to base its use of probability on ineluctable dictates of logic.169 The court itself points out that each defendant's share of the liability in DES cases over time will only be "approximately equivalent to the damages caused by the DES it manufactured."

Damages are spread across those who used identical formulae to manufacture the drug at the time the mother purchased the prescription. This assures that the market figures are percentages of the proper class.170 Liability is shared among the manufacturers rather than placed upon one solely because of that manufacturer's share of the market.171 In short, probability is used in the Sindell case as the logical servant of a principle of law, rather than being used on the theory that a priori principles dictate its use. Once probabilistic tools are understood properly, many, if not all, of the distributive justice problems which attend probabilistic proof can be addressed directly rather than allowed to continue on the unstated and erroneous assumptions that reliance on Bayesian methods minimizes "errors" and maximizes aggregate utility.172

167 Id. at 612-13, 607 P.2d at 937, 163 Cal. Rptr. at 145. The court recognized the problems in defining the market and determining market share. Id. Cf. 2 P. AREEDA & D. TURNER, ANTITRUST LAW §§ 518, 520, 522b, 524 (1978) (discussing analogous problems in defining markets and estimating market share in antitrust law).

168 26 Cal. 3d at 610-11, 607 P.2d at 936, 163 Cal. Rptr. at 145.

169 But see authorities cited note 3 supra, other than those using Bayesian theory heuristically.

170 26 Cal. 3d at 613, 607 P.2d at 937, 163 Cal. Rptr. at 146.

171 See notes 76-82 & accompanying text supra.

172 See notes 82-87 & accompanying text supra.

173 Id. To put the point a slightly different way, one can only meaningfully say an error has taken place in comparison to a rule of law. When the enforcement mechanism substantially alters the rule of law, the enforcement mechanism cannot be said to be minimizing errors in enforcing that rule of law.

174 Id. On the error minimization-utility maximization distinction, see Fisher, Truth as a Problem for Utilitarianism, 89 MIND 249, 255 (1980). Another example of a verification value rule is the best evidence rule, which requires the original of a writing, recording, or photograph to prove the contents thereof, unless failure to produce the original is satisfactorily explained. E.g., FED. R. EVD. 1001-1004. The rule applies even though a copy which has no known errors, but which was not examined against the original to see whether it was an exact copy, is available, so long as the original is available. FED. R. EVD. 1002, 1004. A probabilistic rule would allow the document to be received in evidence if it were a close copy, but not known to be exact. See generally notes 30-33 & accompanying text supra.
BAYESIAN THEORY

SUFFICIENCY OF EVIDENCE, MATHEMATICAL PROBABILITY, AND THE DIRECTED VERDICT TEST

A dictum in *Sargent v. Massachusetts Accident Co.* has proven to be a thorn in the side of those who favor the use of subjective probability analysis in evidence law. In *Sargent*, Judge Lummus stated:

It has been held not enough that mathematically the chances somewhat favor a proposition to be proved. . . . The weight or preponderance of evidence is its power to convince the tribunal which has the determination of the fact, of the actual truth of the proposition to be proved. After the evidence has been weighed, that proposition is proved by a preponderance of the evidence if it is made to appear more likely or probable in the sense that actual belief in its truth, derived from the evidence, exists in the mind or minds of the tribunal notwithstanding any doubts that may still linger there.

This formulation of the preponderance test makes a perception of the standard based on mathematical notions of probability difficult. The *Sargent* test also appears not to square very well with one of the black letter formulations of the preponderance test, that is, whether "the existence of the contested fact is more probable than its non-existence." Moreover, one can infer from the arguments of several commentators that the reasoning in *Sargent* should not preclude subjective probability analysis. Those commentators show that the *Sargent* standard is not as inconsistent with Bayesian analysis as one might suppose. At least one commentator has suggested that it may be superficially consistent with maximizing expected utility. These commentators argue that one would naturally discount a mathematical probability standing alone when there

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166 307 Mass. 246, 29 N.E.2d 825 (1940). The question in *Sargent* was whether plaintiff had sustained his burden of proving that the insured had met his death by accident in showing that the insured embarked on a kayak trip in Northern Canada in September without winter clothing under circumstances discussed in the case which indicate the insured lost his kayak and drowned. No quantified evidence was offered in *Sargent*, nor was any use of subjective probability considered.

167 E.g., M. Finkelstein, supra note 60, at 59-73; Kaye, supra note 86, at 603-11.
168 307 Mass. at 250, 29 N.E.2d at 827.
170 E.g., Tribe, supra note 2, at 1341 n.37, 1349-50 (discussing Smith v. Rapid Transit, Inc., 317 Mass. 469, 58 N.E.2d 754 (1945) (relying on *Sargent*); Kaye, supra note 86, at 610-11. Professors Ellman and Kaye make an argument, basically identical to the one discussed in the text, without discussing *Sargent* or any other case. Ellman & Kaye, supra note 5, at 1158-61. Professor Finkelstein argues that *Sargent* should be assessed on the basis of "equalization of errors." M. Finkelstein, supra note 60, at 69. Because Kaye has effectively refuted Finkelstein's argument, and adopted the argument in the text based on some of Finkelstein's own premises, the text only addresses Kaye's argument. See Kaye, supra note 86, at 605-11.
171 Kaye, supra note 86, at 606. For a general discussion of the utility maximization argument, see notes 86-87 & accompanying text supra.
was no adequate explanation of a failure to produce more particularized evidence.\textsuperscript{172} If in a paternity trial, for instance, the plaintiff showed that seventy percent of all defendants in paternity cases were held liable, and offered no other evidence, the factfinder would be justified in presuming that the defendant in the particular paternity case was not the father as alleged. If the defendant was in fact the father so much particularized evidence would normally be available that the plaintiff's unexplained failure to adduce such evidence should be taken as strongly persuasive in favor of the defendant.\textsuperscript{173}

A second point is that treating a subjective probability based on statistical evidence without particularized proof as insufficient evidence eliminates any incentive to the plaintiff to rely only on statistics. The removal of such an incentive avoids the type of result in the blasting cap case,\textsuperscript{174} in which $M_1$ would have to pay for all damages from defective blasting caps manufactured by a particular process whenever $M_1$ manufactured in excess of half of such blasting caps.\textsuperscript{175}

Relying on these arguments, one can argue that the sufficiency of the evidence test is a standard which minimizes errors along Bayesian lines, promotes the maximum use of particularized proof, and thus reduces inaccuracy. The reduction in inaccuracy results in an increase of expected aggregate utility.\textsuperscript{176}

This twofold argument falls a bit short. An unexplained failure to produce particularized evidence should result in some discounting of what would otherwise be the estimate of the subjective probability; however, quantifying uncertainty is so problematic\textsuperscript{177} that it is questionable, when an estimate, before discounting, is well above .5, say .8, that discounting for uncertainty will result in a subjective probability below .5.

While the second argument\textsuperscript{178} takes some account of the distributive justice problems use of subjective probability may pose,\textsuperscript{179} it falls short of a solution. To the extent particularized nonmathematical evidence is relied on, it necessarily avoids the specific problems of distributive justice which subjective probability poses.\textsuperscript{180} There are, however, two questions

\begin{enumerate}
\item \textsuperscript{172} Ellman & Kaye, supra note 5, at 1159-60; Kaye, Gatecrasher, supra note 29, at 106, 108; Tribe, supra note 2, at 1339 n.33, 1349; Kaye, supra note 86, at 603.
\item \textsuperscript{173} The problem is taken from some discussion in Ellman & Kaye, supra note 5, at 1149-52. The adverse inference from nonproduction of particularized evidence is discussed in authorities cited note 172 supra.
\item \textsuperscript{174} See notes 30-33 & 69-81 & accompanying text supra.
\item \textsuperscript{175} Tribe, supra note 2, at 1349-50; Kaye, supra note 86, at 603-11. For a discussion of the distributive justice implications, see notes 76-87 & accompanying text supra.
\item \textsuperscript{176} This is what Kaye appears to argue. Kaye, supra note 86, at 605. See also note 86 & accompanying text supra.
\item \textsuperscript{177} See notes 29 & 61-66 & accompanying text supra.
\item \textsuperscript{178} See note 175 & accompanying text supra.
\item \textsuperscript{179} See notes 82-87 & accompanying text supra.
\item \textsuperscript{180} Id.
\end{enumerate}
the second argument leaves: first, the test to be applied when there is no particularized evidence which can be submitted, and second, if particularized evidence is submitted, whether probabilistic alternatives should also be admissible.

This article has discussed the first issue above. The issue of whether to allow generalized subjective probabilistic evidence is not one relating to the accuracy of factfinding. Instead, it is an issue of the principle or principles on which rewards and punishments or assets and liabilities are to be meted out; it is an issue of distributive justice. Until this question is resolved, Bayesian theory has no usefulness.

The issue of admissibility when particularized evidence is available is similar. Bayesian theory is useful only if subjective probability estimates are made as to the truth of all material inferences based on the evidence. Quantification of particularized evidence is one of the flaws in Bayesian analysis. Accordingly, courts should take care to use quantified evidence only when either the statistic itself is the subject or, for reasons relating to the underlying substantive rule, the court is willing to accept a statistic as an approximation in that particular case. Bayesian formulae should not be used even as illustrations of the possible effect of quantified evidence. This would apply equally to a chart with “prior” probability estimates based on evidence which had previously been unquantified. For example, the effect of particular “prior” probabilities in conjunction with objective probabilities such as the frequency of a blood trait in the population should not be used. Such an approach would only be useful if subjective probability, or a probability derived on both objective and subjective data, is equivalent to probative force. Such a chart approach is productive only of confusion.

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181 Or when a failure to do so has been adequately explained.
182 See notes 69-87 & accompanying text supra.
183 See notes 187-90 & accompanying text infra (discussion of so-called chart approach).
184 See notes 20-29 & accompanying text supra.
185 See, e.g., note 35 supra.
186 See notes 87-100 supra. See also 2 P. AREEDA & D. TURNER supra note 157, § 507, at 330 (pointing out that all methods of defining market for antitrust purposes "raise difficult problems of proof and judgment"); Brilmayer & Kornhauser, supra note 3, at 121 (discussing usefulness of "summary statistics").
187 See notes 3 & 52 supra; note 190 infra.
188 For a fuller exposition of this chart approach, see Ellman & Kaye, supra note 5, at 1152-58.
189 Professors Ellman and Kaye appear to believe the two are equivalent. Id. at 1157.
190 Professors Ellman and Kaye misstep at least once more in their argument for the chart approach. They argue:

[The modified chart approach] does not ask the jurors to produce [a subjective probability estimate based on unquantified data]. It merely shows them how a correctly ascertained probability would be altered, if one were in fact available. In this way it accurately communicates the significance of the admittedly probabilistic scientific evidence, without requiring the remaining evidence to be expressed as a probability.
Given all these arguments, one still might ask what the word “probable” in the black letter “more probable than not” standard means. By in effect requiring particularized evidence in the case before it, the Sargent court apparently decided that in order to meet the “more probable than not” standard, the party bearing the burden of proof must provide evidence which is persuasive enough that it has an acceptable verification value derived from either in-court testing or extrajudicial experience, regardless of whether an application of mathematical probability theory would yield a probability of greater than .5 for the truth of the proposition at issue. A factor other than mathematical probability must be considered in deciding the degree to which data affect “probability” in evidence terms.

SUBJECTIVE PROBABILITY IN MODELING

Although subjective probability theory is faulty as a device for prescribing factfinding or procedural rules, it does not follow that it is otherwise

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*Id.* at 1157 (footnote deleted). Even assuming that a combination of subjective and objective probabilities has significance, Ellman and Kaye ignore another issue. Their modified chart approach invites juror's to make a subjective probability estimate; otherwise the chart is basically meaningless. That is, it tells the jurors that if they can do X, Y will result. It is doubtful, at best, that jurors can do X (formulate useful subjective probability estimates). Moreover, it is the trial court's duty to decide whether the jury would be justified in drawing certain inferences, for instance, in deciding whether or not to direct a verdict. *See, e.g.*, Pennsylvania R.R. v. Chamberlain, 288 U.S. 333 (1933). Accordingly, Ellman and Kaye's example would be closely analogous to a court instructing a jury that while the court did not know how to decide whether the jury was justified in drawing a specific inference from specific evidence, if such an inference could be properly drawn, then the jury would be justified in reaching a certain conclusion. On its face, that would be a paradigmatic abdication of judicial responsibility. On the whole, Ellman and Kaye's chart approach comes perilously close to begging the question whether a subjective probability can ever be reliable.

*See note 189 & accompanying text supra.* There is no reason, in view of the weaknesses in mathematical subjective probability theory and the lack of identity between Bayesian theory and probative force, to interpret the word “probable” for the purpose of deciding the relevance of evidence any differently from its meaning in the burden-of-proof test as interpreted in this article. *See generally* Fed. R. Evid. 401: “Relevant evidence” means evidence having any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without this evidence.” *See also* note 50 & accompanying text *supra.*

*See note 168 & accompanying text supra.* A contention that “probable” or “probability” used in ordinary conversation is indistinguishable from mathematical notions of “probability” is implausible. Neither *Webster*’s *Third New International Dictionary* 1806 (1976) nor *The Compact Edition of the Oxford English Dictionary* 2309-10 (1971) defines “probable” in terms of mathematics. The Oxford Dictionary does use two mathematical illustrations and both refer to the use of the term “probability” in mathematics. *Id.* Neither purports to confine the meaning of either of these words to the mathematical context. The point here is not that the use of the word “probable” does not imply that mathematical analogies may have some usefulness, but rather that “probable” and “probability” in evidence law are not delimited by mathematical notions. *Cf.* G. *Harman, supra* note 15, at 97-100 (discussing use of one's beliefs in perceiving the definition of words).
useless. It can help illustrate certain kinds of problems. One commentator's discussion of prejudicial evidence is illustrative.\textsuperscript{194} He argues that jurors may regret a decision for party $P$ rather than party $Q$ when that decision is either erroneous or contrary to their sympathy for $Q$ or lack of sympathy for $P$.\textsuperscript{195}

Two examples will illustrate this argument. First, consider the question of the admissibility of the possession of liability insurance to show negligence.\textsuperscript{196} Set aside the question whether the fact that defendant owned such insurance tends to show that defendant was negligent. Even if the evidence tended to show negligence, it should be inadmissible, according to this argument, because it alters the jury's regret matrix. Such evidence would make the jury regret an error in the defendant's favor more than an error in the plaintiff's favor. An error in the plaintiff's favor would seem to impose less hardship, as defendant would have insurance to cover the loss.\textsuperscript{197} In terms of utility, errors in plaintiff's favor would seem to cause less disutility than errors in defendant's favor. Once the balance of disutility between errors in favor of the plaintiff and errors in favor of the defendant is altered, a Bayesian trial model develops a serious flaw.\textsuperscript{198} Therefore, by analogizing the trial process to Bayesian theory, the commentator argues the evidence should be excluded.\textsuperscript{199}

Another use of regret analysis relates to criminal cases.\textsuperscript{200} Suppose there are two defendants, $D_1$ and $D_2$, each accused of committing armed robbery, but not the same armed robbery. Suppose further that other crimes were admissible to show that a defendant acted in conformity with whatever character was shown in those crimes.\textsuperscript{201} If $D_1$ had a prior criminal record, or engaged in illegal acts of which he had not been convicted,\textsuperscript{202} and $D_2$ had no such past, a juror would be more likely to regret erroneously convicting $D_2$ than erroneously convicting $D_1$. Regardless of the probative force of the past record, a juror might regret conviction of $D_1$ less because the marginal stigma of the last of a series of convictions would do less harm to $D_1$ than an initial conviction would do to $D_2$ and because the juror may perceive $D_1$ as likely to have committed or to commit other crimes and thus as a fit subject for punishment.\textsuperscript{203} This change

\textsuperscript{194} Lempert, supra note 3, at 1032-40.

\textsuperscript{195} Id. at 1032-34. While Lempert uses quantified charts, they are not necessary to the discussion here.

\textsuperscript{196} Id. at 1038.

\textsuperscript{197} Id.

\textsuperscript{198} See notes 86-87 & accompanying text supra.

\textsuperscript{199} See Lempert, supra note 3, at 1041.

\textsuperscript{200} Id. at 1039-40. This example is altered slightly from Professor Lempert's version.

\textsuperscript{201} That is, suppose that FED. R. EVID. 404(b) were abolished.

\textsuperscript{202} The change in "regret" may be especially marked in the case of unpunished criminal acts. Lempert, supra note 3, at 1039.

\textsuperscript{203} Id. at 1038-39.
in the juror’s perceived regrets means, in Bayesian analysis, that the burden of proof varies in the prosecutions of $D_1$ and $D_2$. It means that the jurors will be more willing to convict $D_1$ than $D_2$, if one conceives of the burden of proof test as a rule to maximize aggregate utility. The argument is that because application of a different standard of proof to $D_1$ than to $D_2$ is improper, prior crime evidence should be excluded.\footnote{Id. The implications in subjective probability theory of allowing a defendant to prove his good character as a defense, which would alter the jury’s regret matrix in a different way, is a question which Professor Lempert does not address and which will accordingly be reserved for a later time. See generally Fed. R. Evid. 404(a)(1); note 126 & accompanying text supra. Professor Bartels argues that jurors should be informed of the degree of punishment to which the accused, if found guilty, could be subjected, on the theory that the jury should and will increase the burden of proof in a criminal case as the possible punishment increases. Bartels, supra note 86. Without otherwise discussing the merits of Professor Bartels’ argument, it poses an interesting problem when the accused’s prior convictions are relevant to the length of the accused’s possible sentence. If the jury is to be informed of the length of time this particular accused would serve if convicted, then, on Bartels’ reasoning, the burden of proof to convict a defendant with a record of criminal convictions is greater than the burden to convict a defendant without a prior criminal record. On the other hand, if the jury is to be informed of only the standard sentence for a criminal without a prior record, then there is some degree of contradiction of Professor Bartels’ argument that the reasonable doubt standard must be more stringent as the punishment involved becomes more severe, id. at 907.}

The analogy between the rules excluding evidence because it is considered “prejudicial” and shifts in regret matrices unrelated to the facts in issue is a useful one, especially for students who have trouble distinguishing between logical and legal relevancy. Because a rigorous explanation of Bayesian theory and its defects is hardly necessary to explain this point, the analysis is not only an improvement in precision over conclusory statements about prejudice, but a practical improvement as well.\footnote{A modified form of Professor Lempert’s argument worked well for this author in teaching evidence at the University of Miami in 1979-1980. Students several times stated that a regret model, coupled with considerations addressed in the succeeding text, made the distinction between “logical” and “legal” relevance clearer.}

There does seem to be a kind of utility or regret of which the regret analysis does not take account. This is what might be called “macro-disutility,” the disutility with regard to society in general of a change in the evidence rules, rather than with regard to the result in a particular case.\footnote{Although the regret analysis does not consider macro-disutility, the undesired results discussed in the text would not arise under the regret analysis because under this analysis evidence of liability insurance, see text at note 200 supra, and evidence of past crimes, see text at note 204 supra, should be excluded.}

Suppose a rule were adopted allowing proof of $W$’s ownership of liability insurance on the question of $W$’s negligence in manufacturing widgets. If $W$ noticed that $V$, a similarly situated,\footnote{That is, with identical or nearly identical quality control, raw materials, etc.} uninsured widget manufacturer, was held liable less frequently than $W$, or was held liable for lesser damages per capita than $W$ in comparable cases, $W$ might aban-
Bayesian analysis is inadequate as a prescription for rules of factfinding because of defects in its theory of quantification of subjective probabilities, because of unresolvable doubts about the accuracy of Bayesian calculations on account of their lack of susceptibility to testing, and because Bayesian methods can result in the masking or confusion of issues of which incentives, rewards, or penalties the legal system should dispense.

See notes 80-81 & 87 & accompanying text supra.

Cf. Underwood, Law and the Crystal Ball: Predicting Behavior with Statistical Inference and Individualized Judgment, 88 Yale L.J. 1408, 1414-18, 1433-42 (1979) (decisions whether to predict individual behavior and choices of variables on which to predict may involve conflicts with respect for individual autonomy as a social value). Although the thrust of Professor Underwood's argument is slightly different from that in the text, her conclusion is most apposite, when one recalls that a record of prior convictions is, after the individual has been convicted, largely out of the individual's control. "If . . . applicants are regarded as able to change, and especially if certain changes are desirable, then it is preferable to use nonpredictive criteria of selection that tend to induce those changes by reward and punishment, or to use predictive methods that base predictions on factors within the individual's control." Id. at 1448. Another defect in Bayesian theory which one could infer from Professor Underwood's article is that autonomy is a social value which is not readily quantifiable and so may not be subject to restatement in a factfinder's "regret matrix."

See notes 86-87 & accompanying text supra.
questions of distributive justice. These defects can be profitably compared
to a standard of verification value, derivable from the philosophy of science
and from the evidence rules themselves, which seeks to maximize the
testing of data presented to the factfinder. As a nonquantified standard,
it avoids the problems attendant upon Bayesian quantification and inter-
pretation and does not conceal issues of distributive justice.

Bayesian theory can be useful as an illustration of problems in evidence
law. Bayesian theory used for illustrative modeling, as currently
developed, does have limitations analogous, if not identical, to those of
prescriptive Bayesian factfinding theory; however, these limitations are
controllable when Bayesian theory is used only for illustration or limited
analysis. Rather than use as a means of "securing "pseudo-tautological
results" by arguing that a desired result is supported because a
mathematical model, based on questionable assumptions, indicates that
result, Bayesian theorists should concentrate on the primary reason for
mathematical models:

Models are, for the most part, caricatures of reality, but if they
are good, like good caricatures, they portray, though perhaps in
distorted manner, some of the features of the real world.
The main role of models is not so much to explain and to predict—
though ultimately these are the main functions of science—as to
polarize thinking and to pose sharp questions.212

One would have to conclude, given the volume of scholarly discussion,
that Bayesian evidence theorists have succeeded in posing sharp ques-
tions. A Bayesian prescription of factfinding or the rules of evidence,
however, has too many flaws, both practically and theoretically, to be
much more than an academic curiosity.

211 Comment, Mathematical Models of Legal Rules: Application, Exploitation and Inter-